

Investment Distortions and the Value of the Government's Tax Claim

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Abstract

This study integrates the government in the context of company valuation. Our framework allows to analyze and to quantify the risk-sharing effects and conflicts of interest between the government and shareholders when firms follow different financial policies. While -from the government's perspective- the underinvestment problem is well known, we show that firms in some situations invest more than socially desirable depending on the financial policy. Companies that follow a constant leverage policy underinvest. In contrast, firms with fixed future levels of debt overinvest if the gain in tax-shields is big enough to outweigh the loss in the unlevered firm value.

JEL classification: G31, G32, G38, H21, H25

Keywords: corporate tax claim, company valuation, optimal investment, cost of capital

We are grateful to Jan Bartholdy for his valuable comments. This paper has also benefited from the comments of participants at the 2008 European Financial Management Meeting, 2008 Midwest Finance Association Meeting, 2008 Southwestern Finance Meeting, 2008 Portuguese Finance Network Meeting, and the 2008 Financial Management Association European Conference Meeting. The authors gratefully acknowledge funding from the Corporate Finance Seminar, University of Cologne. An earlier version of this paper circulated under the title "Company Valuation, Risk Sharing and the Government's Cost of Capital."

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1 Introduction

This paper analyzes the government's claim in a company and provides a new framework that can be used to quantify the distortion effects of corporate taxes. From a corporate finance view, the government is a claimholder to the company's cash flow, since the government taxes the company's net income to finance public spending. This implies that even though the government provides no capital to the company, the government's claim can be valued similar to shareholder and bondholder claims.¹ However, the government is not a passive bystander. The government's choice of the corporate tax rate is an important factor with respect to the investment decision made by shareholders and it is well known that the existence of corporate taxes distort this investment decision away from the social optimum (Bond and Devereux, 2003). We extend this literature by taking into account that corporate investments spur future growth and that the investment decision depends on the company's financial policy. To this end, we integrate the government into the classical valuation framework and compare the government's and the shareholders' financial position in the company.

A first step toward integrating the government into the valuation process has been taken by Galai (1998). He analyzes the financial position of the government in a one-period setting and derives the tax payments' discount rate. We follow Galai and use the term "government cost of capital" for this discount rate since it is the appropriate rate of return the government requires for holding

¹ This has long been recognized. Domar and Musgrave (1944) state that "By imposing an income tax on the investor, the Treasury appoints itself as his partner". Miller (1988) points out that "The Internal Revenue Service can be considered as just another security holder".

the tax claim.² In this one period setting, Galai also shows that corporate taxes distort company investment decisions.

Since corporations normally exist for many years, Galai's assumptions are quite restrictive. We broaden the analysis to the perpetuity case. Thereby, we are able to analyze the important effects of both growth and the company's financial policy on the government's claim and on the distortion effect of corporate taxes. Being more precise, we study the two most well known financial policies, the fixed debt policy introduced by Modigliani and Miller (1963) and the constant leverage policy analyzed by Miles and Ezzell (1980, 1985). The financial policy is of pivotal importance to the government's claim, since the value of tax shields and, therefore, the value of tax payments depend on the financial policy. Extending the analysis to the growth case, in turn, is essential in order to analyze the distortion effect of taxes on investments, since the growth rate depends on a firm's investment policy.

For the first time we show that corporate overinvestment, i.e., investing more than is socially desirable, is possible and depends on a firm's financial policy. Firms that follow a constant leverage policy never overinvest, but always underinvest. In contrast, for firms with fixed future levels of debt, over- or underinvestment is possible. We show for the first time that a firm may overinvest if the gain in tax-shields is big enough to outweigh the loss in the value of the unlevered firm. Also, if the firm underinvests instead, the severity of underinvestment is less strict than for a constant leverage policy, given the same initial amount of debt.

Our analysis of the government's tax claim and the distortion effect of taxes provides several important insights. First, assessing the risk and value of tax payments becomes increasingly im-

² While Galai (1998) and this paper focus on the government's income by assessing the value and riskiness of the government's tax claim, Novy-Marx and Rauh (2011), for instance, are concerned about the riskiness and the value of government's liabilities.

portant, since a growing number of countries have begun to securitize public assets and borrow against them on the open market. This trend has prompted Eurostat, the statistical office of the European Union, and the Governmental Accounting Standards Board in the US to clarify how to report securitization operations undertaken by the government in national accounts. In this regard, an accurate assessment of the government's tax claim and its riskiness is crucial. Furthermore, by deriving the value of the government's tax claim we contribute to the literature that investigates the components of the government's intertemporal budget to determine whether its fiscal policy is sustainable (Auerbach, 2004). Second, the transparency of the discounted cash flow (DCF) valuation is increased by explicitly integrating the shareholders', the creditors' and the government's perspective into the valuation framework. Third, we illustrate that various policies available to the government to encourage corporate investments toward their socially optimal level should depend on the financial policies pursued by firms and that it is sometimes maybe even necessary to discourage private investment. Finally, our framework allows researchers and the government to estimate welfare losses introduced by corporate taxes thereby providing a useful benchmark when comparing alternative tax codes.

The remainder of this study is organized as follows. Section 2 describes the valuation framework and extends the traditional DCF valuation approach by explicitly modeling the government as a separate claimholder to the company's cash flow stream. Section 3 contains the derivation of the government cost of capital and the value of the government's tax claim. Furthermore, we describe the risk-sharing effects between the government and the stockholders. Section 4 analyzes the conflicts of interest between the shareholders and the government arising from capital budget considerations. Section 5 summarizes the results and discusses implications for future research.

2 Valuation Framework

Our model assumes that the market is complete, frictionless (with the exception of corporate taxes) and all investors have access to all information. In addition, no arbitrage opportunities exist and operating income as well as the fundamental pricing kernel are not affected by the capital structure. Moreover, in order to be consistent with the models of Modigliani and Miller (1963) and Miles and Ezzell (1980, 1985) the government imposes a full-loss-offset tax on net income and we assume a default-free firm.

Since we are analyzing growing perpetuities, we additionally assume a steady-state condition where the balance sheet and income statement items as well as the present value of debt and equity are expected to grow geometrically with the same rate g for all periods $t > 0$. This requires new investments to be financed proportional to net operating profit after tax. Note that this setting is consistent with the well known Gordon growth framework (Gordon and Shapiro, 1956) and therefore guarantees $g = \text{irr} \cdot b$, where irr is the average return on assets (after tax) the unlevered company is expected to earn in every future period and b is the (expected) retention ratio. It is also important to note that irr itself is a function of b , since higher retention ratios lead to decreasing marginal rates of return. Finally, these assumptions imply that the cash flows to each claimant are expected to grow with the rate g (Lundholm and O'Keefe, 2001).

Traditional approaches in company valuation are interested in the *after-tax* value of the firm. It is well known that the after-tax value of a firm, V^L , can always be written as:

$$V^L = E + D = V^U + TS, \tag{1}$$

where

V^L : value of the levered company; the after-tax firm value

E : value of equity

D: value of debt

V^U : value of the unlevered company

TS: value of tax shields

By integrating the government into the valuation process, the company can also be valued on a *before-tax* basis. The before-tax value of a firm, C, can then be written as:

$$C = E + D + G = V^L + G = (V^U + TS) + (G^U - TS) = V^U + G^U, \quad (2)$$

where

G: value of tax payments

G^U : value of tax payments if the firm is unlevered

The before-tax value of the firm from the financing perspective is comprised of the sum of the market values of equity and debt as well as the discounted tax payments. Hence, the before-tax value C unites the views of all three involved claimants, the shareholders', the debt-holders' as well as the government's perspective. This implies that any payments made between the claimants cancel each other out. As a result, the tax shield adds to the value of the unlevered firm and must be subtracted from the present value of taxes in an unlevered firm. Finally, the before-tax value C is equal to the value of an unlevered firm (V^U) which is then corrected for its tax payments by adding G^U . Equation (2) uses the fact that in a perfect capital market, the before-tax value of the company is independent of its capital structure and financial policy, provided that the investment policy is given (Galai, 1998; Goldstein et al., 2001).³ The financing decision does not influence the amount of the before-tax value C, instead it determines how to split the cash flow and, therefore, the value of the company among the three claimants.

³ Otherwise arbitrage opportunities would exist. This proposition is consistent with the Modigliani/Miller Proposition I without taxes, which states that the value of a firm with fixed investments is independent of the distribution of the firm's cash flows.

However, this does not mean that investment and financing decisions of the firm are independent of each other. Since investment decisions are made by the shareholders in order to maximize the value of their invested wealth, capital structure and financial policy should have an influence when determining the optimal investment policy. Therefore, the before-tax value C of the firm implicitly depends on the firm's financing decisions. However, in contrast to the after-tax value of the company, once the investment policy is fixed, a change in financing decisions has no effect on the before-tax value of the company.

Although the capital structure has no direct effect on the company's before-tax cash flow, it determines the way in which this cash flow is distributed among the three claimants. We define the company's free cash flow, FCF, and equity free cash flow, FTE, in the usual way (Arzac and Glosten, 2005):⁴

$$FCF_t = NOPAT_t - NI_t = X_t \cdot (1 - \tau) - NI_t, \quad (3)$$

$$FTE_t = (X_t - r \cdot D_{t-1}) \cdot (1 - \tau) - NI_t - PP_t, \quad (4)$$

where

NOPAT : net operating profits after tax

X : EBIT or earnings before interest expense and taxes

τ : corporate tax rate

NI : net investment, that is, the sum of the change in property, plant and equipment (PPE) and net working capital

r : cost of debt, which is equal to the risk-free rate

PP : principal payments

⁴ Note that free cash flow is defined here as the cash flow of the unlevered company. Furthermore, for simplicity we often suppress the expectation operator $E(\cdot)$ if there is no risk of ambiguity.

The additional cash flows that must be considered in our framework include the company's before-tax cash flow, $CFBT_t$, and the government cash flow if the firm is levered, FTG_t , or unlevered, FTG_t^U , respectively. Following from the cash flow definitions above, this yields:

$$CFBT_t = X_t - NI_t, \quad (5)$$

$$FTG_t = (X_t - r \cdot D_{t-1}) \cdot \tau, \quad (6)$$

$$FTG_t^U = X_t \cdot \tau. \quad (7)$$

Moreover, the representation of the company's cash flow before tax and the free cash flow can be further simplified since new investments must be financed proportional to net operating profit after tax (NOPAT). Taking into account that $NI_t = b \cdot X_t \cdot (1 - \tau)$, the respective representation of (3) and (5) simplify to:

$$FCF_t = X_t \cdot (1 - \tau) \cdot (1 - b), \quad (8)$$

$$CFBT_t = X_t \cdot (1 - b \cdot (1 - \tau)). \quad (9)$$

Equation (9) reveals that company's before-tax cash flow is affected by taxation only through its influence on the net investment level, while the tax payments made by the firm to the government cancel out on a before-tax basis. Based on these insights we define the corresponding valuation equations for the before-tax value of the company and the value of its tax payments:⁵

$$C = \frac{CFBT}{k^C - g}, \quad (10)$$

$$G = \frac{FTG}{k^G - g}, \quad (11)$$

⁵ In order to simplify the notation, time subscripts are usually omitted when referring to the first valuation period $t = 1$. In general, stock variables (e.g., C) are associated with their actual value at the beginning of a period, and flow variables (e.g., $CFBT$) are associated with their expected value at the end of a period.

$$G^U = \frac{FTG^U}{k^{GU} - g}, \quad (12)$$

where

k^C : before-tax cost of capital

k^G : levered government cost of capital

k^{GU} : unlevered government cost of capital

One purpose of this paper is to derive traceable representations for the discount rates in equations (10)-(12) to quantify the risk and the value of tax payments. One important step in this direction is to recognize that k^C is a weighted average of shareholder, bondholder and government cost of capital. This can be seen by taking a closer look at the generation and distribution of cash flows.

By definition the following relation must hold:⁶

$$\begin{aligned} \text{CFBT} &= \underbrace{(k - g) \cdot V^U + (k^{GU} - g) \cdot G^U}_{\text{cashflow generation from assets}} = \underbrace{(k^E - g) \cdot E + (r - g) \cdot D + (k^G - g) \cdot G}_{\text{cashflow distribution to claimants}}, \quad (13) \\ &= \text{FCF} + \text{FTG}^U \quad \quad \quad = \text{FTE} + \text{FTD} + \text{FTG}, \end{aligned}$$

where k is the unlevered cost of equity and FTD is the flow to debt holders (interest and principal payments). Note that CFBT and FTG^U grow with the same rate as FCF since they are proportional to FCF . FTG also grows with the same rate since FTE , FTD , and CFBT grow in a steady state with the same rate. Dividing (13) by C then shows that k^C is a weighted average of shareholder, bondholder and government cost of capital:

$$k^C = k \cdot \frac{V^U}{C} + k^{GU} \cdot \frac{G^U}{C} = k^E \cdot \frac{E}{C} + r \cdot \frac{D}{C} + k^G \cdot \frac{G}{C}. \quad (14)$$

⁶ The fact that the growth rate g must be subtracted from the required rates of return (k^E , r and k^G) becomes obvious if one considers, for example, the value of equity: $E = \text{FTE} / (k^E - g) \Leftrightarrow E \cdot (k^E - g) = \text{FTE}$.

Analogous to the before-tax value of the company, the company's before-tax cost of capital is independent of the firm's financing decision when its investments are fixed. However, the financing decision determines the risk sharing among the three claimants and therefore the weighting scheme in equation (14). Most important to our analysis, the equity and government cost of capital depend on the firm's financial policy.

The two best known financial policies are the fixed debt policy, which is characterized by predetermined and planned future amounts of debt, and the constant leverage policy, for which the debt level is tied to firm value through a constant relation. Due to the definition of the financial policies, for a firm following a fixed debt policy the *level* of debt is known ex-ante, whereas for a firm following a constant leverage policy, the *leverage ratio* is known ex-ante. Independently of the pursued financing strategy we presume a default-free firm throughout the analysis. Although this assumption is often applied in literature (see Penman, 1998), its validity does essentially depend on the stochastic process in the model. Particularly, in case of a fixed debt policy where firm's interest payments on debt grow deterministically with the same rate g as the expected operating profit, this assumption might be critical if the stochastic, operating profit has a sufficiently large downside risk. In order to maintain the no default assumption, we constrain this downside risk by assuming the following EBIT-process:⁷

$$\tilde{X}_t = X_0 \cdot (1+g)^t + \tilde{\varepsilon}_t \text{ with } \tilde{\varepsilon}_t > -(X_0 \cdot (1+g) - r \cdot D) \cdot (1+g)^{t-1} \forall t > 0 \quad (15)$$

In this specification the noise terms $\tilde{\varepsilon}_t$ are independently distributed with a zero mean and in each period EBIT exceeds the interest payments on debt.⁸

⁷ See also Cooper and Nyborg (2006), p. 221. A tilde is used to highlight a random variable.

⁸ We thank an anonymous referee for pointing out that a no default assumption is not trivial when following a fixed debt policy with deterministically increasing debt payments (see also Meulbroek (1999) on this issue). Table 1 in the appendix shows - based on US Compustat data - how critical this assumption is when implementing alternative EBIT-processes which have also been used in the prior literature. Table 1 reports the results of a simu-

Since the financial policy determines the risk of the tax shield and, consequently, the risk of the tax payments, we must address this issue when we derive the government cost of capital and the value of tax payments. Our analysis builds on the textbook formulas for the cost of equity, the weighted average cost of equity (WACC),⁹ and the value of tax shields. In the following, subscript *d* denotes the fixed debt policy, whereas subscript *r* denotes the constant leverage policy.

In the case of a *fixed debt* financial policy, these formulas are:

$$k_d^E = k + (k - r) \cdot \left(1 - \frac{r}{r - g} \cdot \tau \right) \cdot L_d, \quad (16)$$

$$\text{WACC}_d = k - r \cdot \tau \cdot L_d^* \cdot \frac{k - g}{r - g}, \quad (17)$$

$$\text{TS}_d = \frac{\tau \cdot r \cdot D}{r - g} \quad (18)$$

where

$L_d := D / E(\tilde{E})$ and $L_d^* := D / E(\tilde{V}^L)$ (see Kumar, 1975; Bar - Yosef, 1977; Myers, 1977).

In case of a *constant leverage* financial policy, these formulas are specified as:

$$k_r^E = k + (k - r) \cdot \left(1 - \frac{r}{1 + r} \cdot \tau \right) \cdot L_r, \quad (19)$$

$$\text{WACC}_r = k - r \cdot \tau \cdot L_r^* \cdot \frac{1 + k}{1 + r}, \quad (20)$$

lution analysis where we examine if there is a default over a horizon of 500 periods. Overall, we conclude that a no default assumption does not impair our model.

⁹ It is sometimes argued that the WACC approach is not applicable in case of a fixed debt policy when the firm is growing. This is not true, as long as the correct WACC-formula is used. Modigliani and Miller (1963) derive their WACC-formula for the no growth case. If the firm is growing, the WACC-formula need to be adjusted in order to take the growing riskless tax shields into account. For a straightforward derivation see Massari et al. (2008), for the general equivalence of the WACC, APV and Capital Cash Flow valuation methods see Booth (2007). A further, recent evolution of the WACC approach is provided by Molnár and Nyborg (2011) who extend the WACC approach in order to account for personal taxes, risky debt and partial default under a constant leverage policy.

$$TS_r = \frac{r \cdot \tau \cdot L_r^* \cdot V^L}{k - g} \cdot \frac{1 + k}{1 + r}, \quad (21)$$

where

$$L_r := \tilde{D} / \tilde{E} \text{ and } L_r := \tilde{D} / \tilde{V}^L \text{ (see Miles and Ezzell, 1980).}$$

The parameter L denotes the leverage ratio, and L^* denotes the target debt ratio, which could be aspired by the management or influenced by rating agencies.¹⁰ It should also be noted that the definition of the leverage ratio (and henceforth the debt ratio) differs with respect to the presumed financial policy. In case of a constant leverage policy, both future debt levels as well as the future value of equity are stochastic. However, the ratio between future debt and the market value of equity (L_r) is assumed to be deterministic. In contrast, in case of a fixed debt policy, the future debt levels are certain whereas the future market value of equity is stochastic. Then, in accordance with Kumar (1975), we define the leverage ratio (L_d) as the amount of debt divided by the expected market value of equity. For both financial policies, the leverage ratio (debt ratio) is assumed to be constant over time which further implies a time-independent cost of equity (k^E) and time-independent weighted average cost of capital (WACC).

This framework enables us in the subsequent sections to derive the value of the government's tax claim and to analyze the conflicts of interest between the government and the shareholders regarding the company's optimal investment level.

¹⁰ See Graham and Harvey (2001), p. 211 and 234. Nineteen percent of companies in their study state that they do not have an optimal L , whereas 10% have an optimal L ; the rest of the sample firms state that they have a more or less flexible L . Also, Gaud et al. (2005) state that firms generally adjust toward a target debt ratio, however, the speed of adjustment is influenced by institutional factors.

3 The Government's Tax Claim

3.1. Firm's Before-Tax Cost of Capital and Government's Cost of Capital

Before determining the value of government's tax claim, we have to derive the appropriate discount rate for the government's before-tax cash flow. In case of an unlevered firm, the firm makes no principal payments, which implies that taxes paid by the unlevered company (FTG^U) are proportional to FCF:

$$FTG_t^U = \tau \cdot FCF_t / ((1 - \tau) \cdot (1 - b)). \quad (22)$$

Thus, the tax claim and the equity claim have the same risk in the unlevered firm and it follows:¹¹

$$k^{GU} = k. \quad (23)$$

The argument regarding proportional cash flows can be similarly carried over to the before-tax cash flow, since $CFBT_t = (1 - b \cdot (1 - \tau)) \cdot FCF_t / ((1 - \tau) \cdot (1 - b))$. This is consistent with the formal derivation of the firm's before-tax cost of capital by substituting (23) into (14):

$$k^C = k. \quad (24)$$

Hence, since FTG^U , FCF, and CFBT are proportional to each other, they all share the same risk class and have to be discounted at the unlevered cost of equity. Moreover, since we made no assumption regarding the firm's financing decisions, equation (24) holds for every debt ratio and every financial policy.¹²

¹¹ Fernandez (2004) reaches the same conclusion (see equation (10) in Fernandez (2004), p. 148). However, his further analysis of a levered company is flawed, since he does not recognize the principal payment's role. He presents the non-growing perpetuity case in his equation (12) by assuming that independent from the financial policy, no principal payments have to be made. But this is true only for a fixed debt policy (Cooper and Nyborg, 2006, p. 220).

¹² It has to be noted that (23) and (24) and therefore the following results differ in important ways from former conclusions drawn in the one-period-framework (Galai, 1998). This is due to the fact that taxes paid by the unlevered firm are not proportional to FCF in the one-period-framework, since net investment is not proportional to EBIT. Net investment is not proportional to EBIT in the one-period-framework since no cash is spent to finance the capital equipment that wears out. Consequently, net investment solely consists of depreciation, which is assumed to be riskless. In this case, it can be shown for an unlevered firm that the government cost of capital is

Next, the government cost of capital can be generally derived. Independent of firm's particular financial policy, it follows from substituting (24) into (14) after rearranging terms:

$$k^G = k + (k - k^E) \cdot \frac{E}{G} + (k - r) \cdot \frac{D}{G}. \quad (25)$$

While equation (25) generally holds, it is also clear that the government cost of capital must depend on the firm's financial policy. This is inherent in equation (25) since the cost of equity (k^E) is contingent on the firm's financial policy. Therefore, the corresponding representations for the two financial policies can be derived by substituting the respective textbook formula for k^E into the general formula.

In the case of a fixed debt policy, substituting (16) into (25) yields for k_d^G :

$$k_d^G = k + (k - r) \cdot \tau \cdot \left(\frac{r}{r - g} \right) \cdot \frac{D}{G_d}. \quad (26)$$

If the firm instead follows a constant leverage policy, the government cost of capital changes, since the firm's tax shield becomes riskier. This has important implications for the government's claim because the value of taxes naturally depends on the value of tax shields. The government cost of capital in a firm with a constant leverage policy can be derived by substituting (19) into (25):

$$k_r^G = k + (k - r) \cdot \tau \cdot \left(\frac{r}{1 + r} \right) \cdot \frac{D}{G_r}. \quad (27)$$

Note that equation (25) and (26) exhibit a circularity problem, since the value of the tax claim (G) on the right hand side (RHS) of the equation is unknown and the government cost of capital is used to calculate this value. Another obvious outcome of this analysis is the increasing risk of the

typically greater than the equity cost of capital. This implies the before-tax cost of capital to be greater than the unlevered cost of equity.

government's claim when leverage increases which follows from the fact that the government is a residual claim holder, no matter which financial policy the firm pursues.¹³ This raises the question how much risk the government has to take relative to the stockholders and how this relative risk position depends on the firm's financial strategy and growth.

For instance, Fernandez (2004) claims that the government cost of capital in non-growing firms equals the equity cost of equity. However, this is only true for a fixed debt policy, as in this case the tax payments and the equity cash flow are proportional in each period and therefore must exhibit the same risk. This result has also been recognized by several other authors, including Cooper and Nyborg (2006) and Fieten et al. (2005). Fieten et al. (2005) also argue that the government cost of capital in a non-growing default-free firm that follows a constant leverage policy is smaller than the equity cost of capital. This result can be intuitively motivated. Considering a company which switches from a fixed debt to a constant leverage policy while keeping the initial debt value constant, implies that the unlevered firm value (V^U) and the expected tax shield payments remain unchanged. However, the tax shield payments in the future become risky after the first period as they are correlated with the unlevered firm value. This increase of firm's risk position then leads to a higher cost of equity whereas government's risk position decreases ($k_r^G < k_r^E$). While these findings pertain to the no-growth case, it can be shown that the aforementioned results cannot be carried over to the growth case in general. If the firm pursues a fixed debt policy in the growth case, the results differ. In order to derive the corresponding risk sharing effects, we rely on the no-growth case as the benchmark scenario. Then, the shareholders and the government have the same risk position since the tax payments and equity cash flow are proportional

¹³ The effect of leverage on the government's risk position has also been recognized by Modigliani and Miller (1966). They state: "[...] while the government can claim τ per cent of the profits, it must also bear τ per cent of the risk, including the risk introduced by leverage."

($k_d^G = k_d^E$). Due to the necessary net investments and growing debt levels these cash flows are not proportional anymore in the growth case. This drives a wedge between the government and the equity cost of capital. As the value of tax payments, G , is equal to a long position in G^U and a short position in TS , the government's cost of capital is now greater than the equity cost of capital, since growing riskless tax shield payments improve shareholders' risk position at the expense of government's risk position ($k_d^G > k_d^E$).¹⁴ When pursuing a constant leverage policy, the government's cost of capital remains lower than the equity cost of capital in the growth case, at least for most parameter constellations. This can most easily be recognized by analyzing a continuous time setting. In a continuous time setting with continuous adjustments of the debt level future tax shields have to be discounted over all horizons with the business risk rate k (Harris and Pringle, 1985; Arzac and Glosten, 2005). Since $G = G^U - TS$ and the government cost of capital in an unlevered firm equals the business risk rate k , it follows directly for the continuous time setting that $k_r^G = k$. This implies $k_r^G < k_r^E$, independent of the capital structure and growth rate. Since the risk of the governments' claim in the discrete time setting considered by Miles and Ezzell (1980) differs from the continuous time setting only due to the handling of the first period, it follows that the government cost of capital in the discrete time setting is usually only slightly higher than the business risk rate k and therefore for most parameter constellations smaller than the equity cost of capital.¹⁵

3.2. The Firm's Before-Tax Value and the Value of Tax Payments

¹⁴ A formal derivation of the risk sharing effects between the government and the shareholders is available upon request.

¹⁵ For very extreme parameter values it is possible that $k_r^G > k_r^E$.

The former results are used to calculate the firm's before-tax value and the present value of taxes paid by the firm. In section 3.1, it was shown that CFBT is proportional to FCF and therefore $k^C = k$. Hence, the firm's before-tax value is

$$C = \frac{\text{CFBT}}{k-g} = \frac{X \cdot (1-b) \cdot (1-\tau)}{k-g}. \quad (28)$$

Of course, the proportionality in cash flows directly carries over to proportionality in values and the firm's before-tax value can be alternatively expressed as a function of the unlevered firm value:

$$C = V^U \cdot \frac{1-b \cdot (1-\tau)}{(1-b) \cdot (1-\tau)}. \quad (29)$$

Equation (29) shows that the before-tax value of the firm solely depends on its investment policy. That means, given a fixed investment policy, the choice of the debt level and financial policy has no effect on the before-tax value of the firm.

The value of tax payments can now be derived as the difference between the before-tax and after-tax value of the company:

$$\begin{aligned} G &= C - V^L, \\ G &= V^U \cdot \frac{1-b \cdot (1-\tau)}{(1-b) \cdot (1-\tau)} - V^L, \\ G &= \frac{\tau}{(1-b) \cdot (1-\tau)} \cdot V^U - \text{TS}. \end{aligned} \quad (30)$$

In contrast to the before-tax value of the firm, the value of tax payments depends on the debt level and the financial policy through the impact of the tax shield. We now derive a more direct formula for the two financial policies' value of tax payments by substituting the respective tax shield formula into (30) and taking into account that

$$V^U = X \cdot (1-\tau) \cdot (1-b) / (k-g). \quad (31)$$

Thus, in case of a fixed debt policy we substitute (18) into (30) and get after rearranging:

$$G_d = \frac{X \cdot \tau}{k - g} - \frac{r \cdot \tau \cdot D}{r - g}. \quad (32)$$

Note that (32) is equivalent to saying that the value of the tax claim equals the difference between the value of the tax claim in the unlevered firm and the value of tax shields (that is, $G = G^U - TS$).

In case of a constant leverage we substitute (21) into (30). However, we need to consider that the unknown levered firm value (V^L) appears in equation (21). We circumvent this problem by recalling that $V_r^L = C - G_r$ and then solve for the value of tax payments (G_r):

$$G_r = \frac{X \cdot \tau}{k - g} \cdot \frac{(k - g) \cdot (1 + r) - r \cdot L_r^* \cdot (1 + k) \cdot (1 - b \cdot (1 - \tau))}{(k - g) \cdot (1 + r) - r \cdot L_r^* \cdot (1 + k) \cdot \tau}. \quad (33)$$

While the formulas (32) and (33) for the two financial policies look very different, there are still some similarities. The anchor point for the (levered) value of tax payments is in both cases the unlevered value of the tax payments $G^u = (X \cdot \tau)/(k - g)$. In principle, subtracting the value of tax shields leads to the levered value of tax payments. While this is feasible in the fixed debt case, it would not solve the problem in the constant leverage case, since in this case the value of tax shields is not known ex-ante. Therefore, if the firm follows a constant leverage policy the unlevered value of tax payments has to be multiplied by a scalar factor. It is obvious to show that this scalar factor is decreasing in L_r^* , which is consistent with the necessity that leverage decreases the levered value of tax payments.

A closer inspection of the government's claim also shows that the marginal effect of growth on the value of the tax claim is ambiguous. Although higher growth leads to a higher value of the tax claim in the unlevered firm (G^U), it also leads to higher tax-shields (TS). Therefore, since $G = G^U - TS$ and the discount rate for the unlevered tax payments is greater than the tax shield's discount

rate, at some point the negative effect of growing tax shields outweighs the positive effect at the margin and increasing growth lowers the tax claim's value.

Up to this point, we derived the value of the government's tax claim in the growing perpetuity case for the first time. This result is especially important for the securitization of tax revenues and long-term financial planning of the government. The derivation was based on the extension of the well-understood Modigliani-Miller Proposition I to the tax case by incorporating the government as an additional claimholder. In this extension, the before-tax value of the company is independent of its capital structure when its investments are fixed and is comprised of the sum of the values of the three claim holders. This framework also allowed us to derive the company's before tax value which will be of special interest in the next section where we compare the government's and the stockholders' view regarding the optimal investment level conditional on the company's financial policy. In appendix 1 we provide two numerical, comprehensive examples which calculate the government's as well as the shareholders' risk position and claims for a fixed debt policy (example 1) and a constant leverage policy (example 2). Consistent to our model assumptions the examples presume no default, that is, the debt liabilities are always redeemed and therefore they have no systematic risk ($\beta_D = 0$).

4 Conflict of Interest Arising from Capital Budgeting: Corporate Under- and Overinvestment

The stockholders' objective is to maximize the *after-tax* value of the firm (V^L).¹⁶ This implies that stockholders, who are in charge of investment decisions, will undertake all projects that add

¹⁶ We assume that the potential conflicts of interest between bondholders and stockholders are solved through costless negotiation and side payments.

to the after-tax value of the firm. The government's objective, however, should be to maximize total welfare, which would be achieved by maximizing the *before-tax* value of the firm (C). In a perfect capital market without taxes, these two objectives would yield the same optimal investment level. However, in a world with corporate taxes these two objectives are not the same, and we will show that the optimal investment plans of the government and stockholders usually differ. In this respect, Galai (1998) has analyzed a one-period framework, finding that stockholders always invest less than is socially desirable. In the perpetuity case, we also find that corporate underinvestment is a possible case. However, we show that in some situations, rational stockholders invest more than is socially desirable.

Before we begin to analyze corporate under- and overinvestment, we take a closer look at the stockholder's capital budget decision within the Gordon growth framework. Since net investment is solely financed by retained NOPAT, the stockholder capital budget problem is to find the retention ratio b that maximizes V^L .¹⁷ In order to find a unique and economically plausible solution within the Gordon growth framework, the average internal rate of return (irr) must be modeled as a function of the retention ratio (b). Economically, it is reasonable to assume diminishing returns in b ($\partial irr / \partial b < 0$). Furthermore, the Gordon growth framework implicitly assumes that this investment opportunity set (IOS) – that means the functional relation between irr and b – is expected to be invariant over time.¹⁸

¹⁷ One could expect that new investments are financed from retained earnings instead of NOPAT. However, note that the analysis focuses on the entity approach and such NOPAT is the appropriate choice. Outside financing can be introduced but does not change the economic implications of the model (Gordon and Gould, 1978). Additionally, the *actual* retention ratio does not need to be the same in all future periods; only the *expected* retention ratio must be time invariant. Moreover, by assumption, the retention ratio must not be correlated with the pricing kernel to rule out further pricing implications.

¹⁸ See Gordon and Shapiro (1956) and Lintner (1963). An in depth analysis of this assumption can be found in Elton and Gruber (1976).

Additionally, since optimal investment is decided at the margin, it is necessary to define the marginal rate of return (irr'). An investment's marginal return is defined in the usual way as the partial derivative of the total dollar return with respect to the quantity of funds invested. The quantity of funds invested at the valuation date (NI_t) in turn establishes the current and expected future retention ratio ($b = NI_t/NOPAT_t$). The investment marginal rate of return is therefore defined as:¹⁹

$$irr' = \frac{\partial(NI \cdot irr(NI/NOPAT))}{\partial NI} = irr + NI \cdot \frac{\partial irr}{\partial(NI/NOPAT)} \cdot \frac{\partial(NI/NOPAT)}{\partial NI} = irr + b \cdot \frac{\partial irr}{\partial b}. \quad (34)$$

In addition, Lintner (1964) shows that this marginal rate of return also equals the partial derivative of growth with respect to the retention ratio:

$$irr' = irr + b \cdot \frac{\partial irr}{\partial b} = \frac{\partial g}{\partial b} > 0. \quad (35)$$

Hence, increasing net investments by choosing a higher retention ratio b leads to a higher growth rate g . However, since irr is a decreasing function of b , irr' and $\partial g / \partial b$ are decreasing functions of b as well. Moreover, the marginal rate of return is always smaller than the average rate of return ($irr' < irr$). Fig. 1 graphs the time-invariant IOS; the average and marginal returns are arbitrarily specified as linear functions in b .

--- Insert Fig. 1 about here ---

In this setting, a conflict of interest between stockholders and the government arises if the optimal retention ratio from the standpoint of the stockholders does not equal the optimal retention

¹⁹ In the following analysis, the notation is simplified by not explicitly stating that irr and irr' are functions of b .

ratio the government would choose if it were in charge of the investment decision. Economic intuition tells this situation is the usual case, since different levels of debt and different financial policies lead to different investment decisions by the stockholders, whereas the government's optimal retention ratio is independent of the company's capital structure and financial policy.

We therefore have to analyze for both financial policies which retention ratio maximizes V^L (the stockholders' objective) and which retention ratio maximizes C (the government's objective).

This can be analyzed by taking the partial derivative of V^L and C with respect to b .

4.1. The Stockholders' Optimal Investment Policy

First, we derive a relation that ensures the maximization of stockholders' wealth for an unlevered firm. By noting that g is an increasing function of the retention ratio b , the unlevered firm value is maximized with respect to b when (recall equation (31)):

$$\frac{\partial V^U}{\partial b} = \frac{\partial \left(\frac{X \cdot (1-b) \cdot (1-\tau)}{k-g(b)} \right)}{\partial b} = \frac{X \cdot (1-\tau) \cdot \left((1-b) \frac{\partial g}{\partial b} - (k-g) \right)}{(k-g)^2} = 0. \quad (36)$$

Solving for $\text{irr}' = \partial g / \partial b$, V^U is maximized, when b is set such that:

$$\text{irr}' = \frac{k-g}{1-b} = \frac{k-b \cdot \text{irr}}{1-b}. \quad (37)$$

The left hand side (LHS) of (37) is the marginal rate of return on investment which is a decreasing function of b . We call the RHS the hurdle rate (HR) function, since as long as the marginal rate of return (or the marginal growth rate respectively) is greater than the value of the HR function, additional profitable investments can be undertaken by increasing the retention ratio. The optimal retention is reached where the marginal rate of return function intersects with the HR function. The HR function considers the advantage versus the disadvantage of increasing growth as it depends itself on the retention b . The denominator of the HR function is the payout ratio

which determines the fraction of NOPAT which is available to stockholders as free cash flows. The higher is the retention ratio b , the more the stockholders have to give up current cash flows in order to finance net investments. On the other hand, the stockholders are compensated by an accelerated growth of the free cash flows which is captured by a lower perpetuity discount factor $(k-g)$. The unlevered cost of equity in the denominator of the RHS of (37) could also be a function of growth and therefore of b . However, for simplicity and in line with Kumar (1975, p. 536) we assume that k is independent of growth. This essentially means that additional investments do not change the risk class of the firm.

Next, we turn to the levered case. The after-tax firm can be generally written as:

$$V^L = \frac{X \cdot (1-b) \cdot (1-\tau)}{WACC(b) - g(b)}. \quad (38)$$

The partial derivative with respect to b is:

$$\frac{\partial V^L}{\partial b} = \frac{X \cdot (1-\tau)}{(WACC(b) - g(b))^2} \cdot (g(b) - WACC(b) - (1-b) \cdot (\partial WACC(b) / \partial b - \partial g / \partial b)). \quad (39)$$

Setting (39) equal to zero while noting that $\partial g / \partial b = irr'$, we find that V^L is maximized when b is set such that:

$$irr' = \frac{\partial g}{\partial b} = \frac{WACC(b) - b \cdot irr}{1-b} + \frac{\partial WACC(b)}{\partial b}. \quad (40)$$

The LHS of (40) is again the marginal rate of return on investment which is a decreasing function of b whereas the RHS is the HR function for a levered firm which is valid for both financial policies.

The critical question is how the HR function depends on b . If WACC is independent of b , the HR function first falls and then rises as b increases. However, the WACC might depend on b , as higher growth implies higher tax shields, and depending on their risk level, these tax shields

might change the WACC of the company. Here lies the crucial difference between the two financial policies. We know that growth has no impact on the WACC if the firm follows a constant leverage policy (see (20)). However, the WACC of a firm following a fixed debt policy depends on the growth rate (see (17)).

First, we analyze the case where the firm follows a constant leverage policy. This is easily done, since $\partial WACC_r / \partial b = 0$. The optimal retention ratio, b_r^* , is reached when

$$irr' = \frac{\partial g}{\partial b} = \frac{WACC_r - b \cdot irr}{1 - b}. \quad (41)$$

At the optimum, it can be easily shown that (Lintner, 1963):

$$irr > WACC_r > irr'.$$

At first glance, this result seems to contradict traditional finance theory which states that investors should only undertake investments with a marginal rate of return greater than the weighted average cost of capital. However, it is essential to note that the definition used in the Gordon growth framework for the *investment* marginal rate of return is not equal to the *investors'* marginal return (Lintner, 1963). This is because defining the average rate of return as a time-invariant function of b implies that additional investment with positive marginal return increases the next period's cash flow, leading to higher dollar amounts of retention that are reinvested at the time-invariant *average* rate of return. Thus, the *investors'* marginal return definition must include these additional returns generated by future (re)investments. Put differently, it can pay off to undertake investments with $WACC > irr'$ because additional returns can be generated on future investments due to growing retentions that are expected to earn the average rate of return each year with $irr > WACC$.

We now turn to the case where the firm follows a fixed debt policy. Unfortunately, the HR function for a firm following a fixed debt policy is less tractable, since WACC depends on b . The

HR function is a polynomial of order three in b .²⁰ The resulting maximization condition for shareholders' optimal retention ratio b_d^* is:

$$\text{irr}' = \frac{X \cdot (r - b \cdot \text{irr})^2 \cdot (k - b \cdot \text{irr}) \cdot (1 - \tau)}{A \cdot X + \tau \cdot r \cdot D \cdot (k - b \cdot \text{irr})^2}, \quad (42)$$

with

$$A = (\tau - 1) \cdot b^3 \cdot \text{irr}^2 + ((1 - \tau) \cdot (\text{irr} + 2 \cdot r)) \cdot \text{irr} \cdot b^2 + ((\tau - 1) \cdot (r + 2 \cdot \text{irr})) \cdot b \cdot r + (1 - \tau) \cdot r^2.$$

Comparing the functional form of (41) and (42), it is obvious that the optimal retention ratio is not the same for both financial policies, even if the same level of debt (or leverage ratio) is chosen by the stockholders.

4.2. The Government's Optimal Investment Policy

The next step is to derive the equivalent maximization condition for the government. First, we consider the following expression:

$$C = \frac{X \cdot (1 - b \cdot (1 - \tau))}{k - g(b)}$$

The before-tax value C is then differentiated with respect to b :

$$\frac{\partial C}{\partial b} = \frac{X}{(k - g(b))^2} \cdot ((1 - \tau) \cdot (g(b) - k) + (1 - b \cdot (1 - \tau)) \cdot \partial g / \partial b). \quad (43)$$

Setting (43) equal to zero and solving for $\text{irr}' (= \partial g / \partial b)$ yields:

$$\text{irr}' = \frac{(1 - \tau) \cdot (k - b \cdot \text{irr})}{1 - b \cdot (1 - \tau)}. \quad (44)$$

²⁰ This can be verified by taking the partial derivative of $V^L = X \cdot (1 - \tau) \cdot (1 - b) / (k - g(b)) + \tau \cdot r \cdot D / (r - g(b))$ with respect to b . After solving the resulting first-order condition for irr' , we derive (42).

In the case of no growth (that is, $b = 0$), the value of the government's HR function is $k \cdot (1 - \tau)$. Note that for any admissible level of debt and for both financial policies, this value is smaller than the WACC in the no-growth case. This means that with respect to the constant leverage policy, the HR function of the government starts below the HR function of the stockholders.²¹

4.3. The Distortion Effects of Taxes on Corporate Investment

The obvious difference between the government's and stockholders' optimal investment level shows that the existence of corporate taxes has a distorting effect on the company's investment level. Moreover, these distortions seem to depend on the firm's financial policy.

First, we study whether stockholders invest less than is socially desirable. Stockholders underinvest if their optimal retention ratio is smaller than the government's optimal retention ratio. If the government HR function were below the stockholder HR function for all admissible configurations of the relevant valuation parameters, the government would benefit more from growth than the stockholders and stockholders would always underinvest.

4.3.1. Case 1: An Unlevered Company

One obvious case in which stockholders underinvest is the case of an unlevered company. This can be seen by comparing the HR functions of the government and the stockholders:

$$\frac{(1 - \tau) \cdot (k - b \cdot irr)}{1 - b \cdot (1 - \tau)} < \frac{k - b \cdot irr}{1 - b}. \quad (45)$$

The LHS of (45) is the government HR function, while the RHS is the stockholder HR function for an unlevered company. The statement in equation (45) is true for all admissible values of

²¹ The same does not directly follow for the fixed debt policy because $\partial WACC_d / \partial b \neq 0$ at $b = 0$; see (40).

k , b , irr , and τ . Thus, the government chooses a higher retention ratio to induce a higher growth rate g . The reasoning for the underinvestment of unlevered companies is straightforward: If the firm is completely equity-financed, the stockholders intend to maximize the unlevered firm value V^U by choosing the appropriate retention ratio b_U^* . However, the government intends to maximize the before-tax value C which is equal to the sum of V^U and G^U . As the value of unlevered tax payments (G^U) is a strictly increasing function of growth, G^U adds to the before-tax value of the company and social welfare is optimized, when the government chooses a retention ratio b_G^* with $b_G^* > b_U^*$. Hence, the government's marginal gains of growth are greater than the stockholders' marginal gains for all retention ratios, that is, the government HR function is always below the stockholder HR function. Thus, stockholders always underinvest if the firm is unlevered.

4.3.2. *Case 2: A Levered Firm*

In this section we analyze how a constant leverage policy or a fixed debt policy, respectively, influences the investment distortions. We show that the option for debt financing leads to ambiguous investment distortions. For some admissible parameter configurations, we observe the same finding, which has already been documented in the literature (Galai, 1998; Bond and Devereux, 2003), that is, stockholders of levered companies invest less than is socially desirable. However, the magnitude of underinvestment depends on the financial policy, since different financial policies lead to different optimal retention ratios for stockholders. Furthermore, we show that investors may choose a higher investment level than the government, that is, investors may also overinvest.

Pursuing a constant leverage policy shareholders always underinvest, since the government's hurdle rate is always below the stockholders' hurdle rate. In order to get this result, we first identify the retention ratio \bar{b}_r for which the government and the stockholder HR functions intersect.

This happens if

$$\frac{WACC_r - b \cdot irr}{1 - b} = \frac{(1 - \tau) \cdot (k - b \cdot irr)}{1 - b \cdot (1 - \tau)}.$$

Solving for b yields:

$$\bar{b}_r = \frac{WACC_r - (1 - \tau) \cdot k}{(1 - \tau) \cdot (WACC_r - k) + \tau \cdot irr}. \quad (46)$$

While the government maximizes $C = V^U + G^U$, the shareholders optimize the after-tax firm value $V^L = V^U + TS$. Keeping in mind that the government HR function starts below the stockholder HR function if the firm follows a constant leverage policy, then for $b < \bar{b}_r$ the government's marginal gain in G^U from increasing b outweighs the stockholders' marginal gain in TS . If the marginal rate of return irr' intersects the government's and the stockholders' hurdle rates at $b = \bar{b}_r$, the marginal gain in G^U is equal to the marginal gain in TS and C equals V^L . It follows that for any $b > \bar{b}_r$: $C(b) - V_r^L(b) < 0$. However, by assumption, the before-tax value of the firm cannot be smaller than the after-tax value. Otherwise, we would have $G_r < 0$.

In contrast, when the firm pursues a fixed debt policy, we show for the first time that the stockholders may choose a higher investment level than the government. Again, the shape and the location of the government HR versus the shareholders HR function determines if there is an under- or overinvestment scenario. Unfortunately, shareholders' HR function (42) in case of a fixed debt policy is rather complex and not easy to interpret. This complexity arises because firm's tax shield payments are riskless and henceforth discounted at a different rate than the firm's free cash

flows. But the economics which cause the shape of the shareholder HR function can be formally illustrated.

Again, the government's objective is to maximize the before-tax value $C = V^U + G^U$ whereas shareholders optimize the after-tax value of the firm $V^L = V^U + TS$. An overinvestment scenario therefore occurs if at $b = b_G^*$ the marginal gain in the after tax value by increasing growth is greater compared to the marginal gain in the before tax value:

$$\left[\frac{\partial V^U}{\partial b} + \frac{\tau \cdot r \cdot D}{(r-g)^2} \cdot \frac{\partial g}{\partial b} \right]_{b=b_G^*} > \left[\frac{\partial V^U}{\partial b} + \frac{\tau \cdot X}{(k-g)^2} \cdot \frac{\partial g}{\partial b} \right]_{b=b_G^*} = 0. \quad (47)$$

This is equivalent to saying that the marginal gain in the after tax value has to be positive since at $b = b_G^*$ the marginal gain in the before tax value is zero, because the government has optimally balanced the loss in V^U with a corresponding gain in G^U . If firm's marginal gain in TS_d is still greater than the marginal loss in V^U at $b = b_G^*$, the firm will choose a higher investment level than the government. Specifically, two circumstances imply an overinvestment scenario. First, the shareholders have to choose a sufficiently high debt level to increase the initial level of tax shield payments. Second, the IOS must provide a productive set of growth opportunities, such that the government itself prefers a high growth rate. Then, at $b = b_G^*$ shareholders still benefit from increasing growth as the marginal gain in TS outweighs the marginal loss in V^U (see equation (47)). Formally, the overinvestment scenario is characterized by:

$$g(b_G^*) > r + \frac{k-r}{1 - \sqrt{\frac{X}{r \cdot D}}} =: \underline{g} \quad (48)$$

In situations where condition (48) is not satisfied, shareholders will underinvest as in the case of a constant leverage policy. However, it can further be shown that the issue of underinvestment is

always more severe in case of a constant leverage policy, that is, we have $(b_d^* > b_r^*)$ given the same initial amount of debt. This is due to the fact that $TS_d > TS_r$ for the same level of debt.

4.3.3. Numerical Illustration

In a next step, we present a simple numerical illustration with a specific linear IOS. Using this IOS in conjunction with the derived hurdles rates, we present a simple example in which the stockholders choose (i) an optimal retention ratio $(b_{d,o}^*)$ that is *higher* than the government's optimal retention ratio (b_G^*) for a fixed debt policy, (ii) an optimal retention ratio $(b_{d,u}^*)$ that is *smaller* than the government's optimal retention ratio (b_G^*) for a fixed debt policy, and finally (iii) where the firm chooses an optimal retention ratio (b_r^*) that is smaller than the government's optimal retention ratio (b_G^*) for a constant leverage policy.

As a first step, we have to define an explicit function for the IOS. For simplification, we assume a linear form (see Lerner and Carleton 1964):

$$\text{irr} = a - q \cdot b. \tag{49}$$

We demand that $r > a - q$ and $a > 2 \cdot q$. These conditions simply imply that the average rate of return falls below the cost of debt with 100% retention, while the marginal rate is greater than zero at that point. The corresponding marginal rate of return function is:

$$\text{irr}' = \text{irr} + b \cdot \frac{\partial \text{irr}}{\partial b} = a - 2 \cdot q \cdot b. \tag{50}$$

Explicitly modeling the irr' function enables us to determine the optimal retention ratio for the government. Substituting (50) into (44) and solving for b yields the government's optimal retention ratio (b_G^*):²²

$$b_G^* = \frac{1 - \sqrt{1 - \frac{(a - k \cdot (1 - \tau)) \cdot (1 - \tau)}{q}}}{1 - \tau}. \quad (51)$$

Now, after deriving the government's optimal retention ratio, we first have to choose admissible parameter values that imply corporate overinvestment for a fixed debt policy. We choose $X = 400.00$, $\tau = 0.400$, $k = 0.100$, $r = 0.065$ and $D = 1,400.00$ according to example 1 and additionally assume an IOS with $a = 0.105$ and $q = 0.050$. Substituting the relevant values into (51) yields the government's optimal retention ratio at $b_G^* = 0.536$. Analogously, solving (42) for b yields the stockholders' optimal retention ratio at $b_{d,o}^* = 0.873$.²³ Second, we alternatively assume a different initial debt level ($D = 791.58$) that leads to underinvestment for a fixed debt policy ($b_{d,u}^* = 0.362$). Finally, turning to the constant leverage policy, we choose the same debt level $D = 791.58$ as in the underinvestment scenario for the fixed debt policy which implies a $WACC = 0.092$ according to example 2. Then, the shareholders' hurdle rate (41) induces an op-

²² It is important to note that the government's optimal retention ratio depends on the tax rate. This follows from the fact that the retention is defined with respect to NOPAT, which also depends on the tax rate. Moreover, the IOS parameters a and q also depend on the tax rate since irr is the internal rate of return after tax. However, the government's optimal *level* of investment ($NI_G^* = NOPAT \cdot b_G^* = X \cdot (1 - \tau) \cdot b_G^*$) does not depend on the tax rate. This result stems from government's calculus to maximize the before-tax value C which does not depend on taxes. Hence, government's optimal chosen level of net investments is also independent of taxes. This can be easily verified as government's maximization problem can equivalently formulated on a before-tax basis with a before-tax retention ratio b_{BT} and a before-tax internal rate of return irr_{BT} . Then, both the optimal before-tax retention ratio b_{BT}^* and the optimal level of net investments NI_G^* are required to be independent of taxes. A formal derivation is available upon request.

²³ All other solutions of b lie outside the admissible parameter space; four of them are complex, and the fifth equals 2.281.

timal retention ratio $b_r^* = 0.140$. Furthermore, equating the marginal rate of return irr' with the HR function of an unlevered firm (compare the RHS in equation (45)) and solving for b yields the optimal retention ratio $b_U^* = 0.051$ for an unlevered firm. Figure 2 illustrates these effects.

--- Insert Fig. 2 about here ---

The intersection of the marginal rate of return irr' with the corresponding HR functions each characterizes an optimal retention ratio. The example shows that the firm underinvests - when pursuing a constant leverage policy - whereas for a fixed debt policy the firm may under- or over-invest. The example reveals that for the same initial amount of debt the underinvestment problem is less severe for the fixed debt case than for the constant leverage case:

$$b_U^* = 0.051 < b_r^* = 0.140 < b_{d,u}^* = 0.362 < b_G^* = 0.536 < b_{d,o}^* = 0.873. \quad (52)$$

--- Insert Table 2 about here ---

Table 2 provides some more insights into the welfare losses associated with the overinvestment scenario. It shows that retaining 53.6% of NOPAT maximizes the before-tax value of the firm, yielding a before-tax value of 4,671.79 and an after-tax value of 3,494.29. By increasing the retention ratio to 87.3%, the shareholders decrease the before-tax value to 4,100.69, but they ultimately gain by increasing the after-tax value to 3,837.73. Hence, stockholders invest more than is socially desirable, since the gain in tax shields $(3,182.53 - 1,577.77 = 1,604.76)$ can outweigh the loss in the value of the unlevered company $(655.20 - 1,916.52 = -1,261.32)$. As a

consequence, the welfare loss (WL) due to the investment decision made by the shareholders amounts to 571.10 (690.22-1,261.32).

Summing up, the main finding of this section is that corporate overinvestment is possible, depending on the financial policy. Firms that follow a constant leverage policy never overinvest, but always underinvest. Firms with fixed future levels of debt may overinvest if the gain in tax-shields is big enough to outweigh the loss in V^U .

5 Conclusion

This study shows that treating the government's tax claim explicitly in company valuation provides several advantages over the usual approach of only implicitly correcting the cost of capital or cash flows for the tax burden. First, starting from the Modigliani-Miller framework without taxes, it is apparent that after introducing the tax authority, the value of the before-tax cash flow available to all three parties is independent of its distribution among the three claimants. Value additivity is retained. The basic knowledge that is valid in the no-tax case can be transferred to the tax case. Given a fixed investment plan only the distribution (rather than the creation) of value is affected by the introduction of corporate taxes.

Assuming either a fixed or a proportional debt policy, the cost of capital for the government is derived, and the present value of the government's claim in the case of a (growing) perpetuity is explicitly calculated. The risk position of the government is analyzed, and it is shown that risk-bearing depends on the company's chosen financial policy. In a positive growth (no-growth) setting, the government's risk is greater than (equals) the equity cost of capital if the firm follows a fixed debt policy. In turn, if the firm follows a constant leverage policy, the government's risk is smaller than the equity cost of capital.

Building on the derivation of the company's before-tax value, we show that multiple conflicts of interest exist between the stockholders and the government. Stockholders usually invest less than is socially desirable. However, for the first time, we show that corporations might even invest more than is socially desirable if the value of tax shields is strongly increased. Moreover, the possibility of corporate overinvestment depends on the financial policy. Firms that follow a constant leverage policy never overinvest, but always underinvest. Firms with fixed future levels of debt may overinvest if the gain in tax-shields is large enough to outweigh the loss in the value of the unlevered company. Also, if firms underinvest instead when pursuing a fixed debt policy, it can be shown that the chosen investment level outweighs the level, the shareholders would choose in case of a constant leverage policy given the same initial amount of debt. These conclusions show that the government should take the financial policy of the company into account when encouraging corporate investment by using, e.g., investment tax credits.

In general, this framework can be used to analyze whether the tax system is neutral with respect to a firm's investment policy. Further research may also integrate personal taxes into the analysis. In addition, the efficiency of government remedies, such as investment tax credits and subsidy rates, in encouraging investment policies toward the social optimum may be studied. Moreover, this framework can be used to calculate the elasticity of investment with respect to the corporate tax rate and quantify the welfare losses that arise from corporate taxation. An empirical examination would need to quantify the effective marginal and average tax rates and derive a realistic function of the investment opportunity set. However, when using this framework, all these studies should be based on the rationale that the social optimal investment policy and, therefore, the social optimal before tax value of the company are independent of a firm's financial policy.

Appendices

Appendix 1 Valuation Examples

Example 1: Valuation of a Hypothetical Firm Following a Fixed Debt Policy

Assumptions:		g=0%	g=5%
Investment ratio	b	0.00%	50.00%
Internal rate of return	irr	-	10.00%
Risk-free rate	r_f	6.50%	6.50%
Unlevered beta	β^U	1	1
Equity premium	p	3.50%	3.50%
Beta of debt	β^D	0	0
Tax rate	τ	40%	40%
Initial EBIT	X	\$400	\$400
Initial level of debt	D	\$1400	\$1400
Implied input factors:			
Growth	$g = b \cdot irr$	0.00%	5.00%
Unlevered cost of equity	$k = r_f + p \cdot \beta^U$	10.00%	10.00%
Cost of debt	$r = r_f + p \cdot \beta^D$	6.50%	6.50%
Net investment	$NI = b \cdot X \cdot (1 - \tau)$	\$0	\$120
Initial free cash flow	$FCF = X \cdot (1 - \tau) \cdot (1 - b)$	\$240	\$120
Initial free taxes	$FTG^U = X \cdot \tau$	\$160	\$160
Before-tax cash flow	$CFBT = X - NI$	\$400	\$280
Valuation and CoC:			
Gross value of firm	$C = CFBT / (k - g)$	\$4,000	\$5,600
Value of unlevered firm	$V^U = FCF / (k - g)$	\$2,400	\$2,400
Value of unlevered taxes	$G^U = FTG^U / (k - g)$	\$1,600	\$3,200
Value of tax shields	$TS_d = r \cdot \tau \cdot D / (r - g)$	\$560	\$2,426.67
Value of taxes	$G_d = G^U - TS_d$	\$1,040	\$773.33
Value of levered firm	$V_d^L = C - G_d$	\$2,960	\$4,826.67
Equity value	$E = C - G_d - D$	\$1,560	\$3,426.67
Leverage ratio	$L_d = D / E(\tilde{E})$	89.74%	40.86%
Beta of government	$\beta_d^G = \beta^U + (\beta^U - \beta^D) \cdot \tau \cdot (r / (r - g)) \cdot (D / G_d)$	1.5385	4.1379
Beta of equity	$\beta_d^{EK} = \beta^U + (\beta^U - \beta^D) \cdot (1 - r / (r - g)) \cdot \tau \cdot L_d$	1.5385	0.7004
Gov. cost of capital	$k_d^G = k + (k - r) \cdot \tau \cdot (r / (r - g)) \cdot D / G_d$	11.88%	20.98%
Equity cost of capital	$k_d^E = k + (k - r) \cdot (1 - (r / (r - g)) \cdot \tau) \cdot L_d$	11.88%	8.95%
WACC	$WACC_d = k \cdot \left[1 + \left(\frac{g}{k} - 1 \right) \cdot \left(\tau \cdot \frac{D}{V^L} \cdot \frac{r}{r - g} \right) \right]$	8.108%	7.486%

Example 2: Valuation of a Hypothetical Firm Maintaining a Constant Leverage Ratio

Assumptions:		g = 0%	g = 5%
Investment ratio	b	0.00%	50.00%
Internal rate of return	irr	-	10%
Risk-free rate	r_f	6.50%	6.50%
Unlevered beta	β^U	1	1
Equity premium	p	3.50%	3.50%
Beta of debt	β^D	0	0
Tax rate	τ	40%	40%
Initial EBIT	X	\$400	\$400
Target leverage ratio	$L_r^* = \tilde{D} / \tilde{V}^L$	30%	30%
Implied input factors:			
Growth	$g = b \cdot irr$	0.00%	5.00%
Leverage ratio	$L_r = L_r^* / (1 - L_r^*)$	42.86%	42.86%
Unlevered cost of equity	$k = r_f + p \cdot \beta^U$	10.00%	10.00%
Cost of debt	$r = r_f + p \cdot \beta^D$	6.50%	6.50%
Net investment	$NI = b \cdot X \cdot (1 - \tau)$	\$0	\$120
Initial free cash flow	$FCF = X \cdot (1 - \tau) \cdot (1 - b)$	\$240	\$120
Initial free taxes	$FTG^U = X \cdot \tau$	\$160	\$160
Before-tax cash flow	$CFBT = X - NI$	\$400	\$280
Valuation and CoC:			
Gross value of firm	$C = CFBT / (k - g)$	\$4,000	\$5,600
Value of unlevered firm	$V^U = FCF / (k - g)$	\$2,400	\$2,400
Value of unlevered taxes	$G^U = FTG^U / (k - g)$	\$1,600	\$3,200
Value of taxes	$G_r = \frac{X \cdot \tau \cdot (k - g) \cdot (1 + r) - r \cdot L_r^* \cdot (1 + k) \cdot (1 - b \cdot (1 - \tau))}{k - g \cdot (k - g) \cdot (1 + r) - r \cdot L_r^* \cdot (1 + k) \cdot \tau}$	\$1,389.71	\$2,739.02
Value of levered firm	$V_r^L = C - G_r$	\$2,610.29	\$2,860.98
Value of tax shields	$TS_r = \frac{r \cdot \tau \cdot L_r^* \cdot V^L}{k - g} \cdot \frac{1 + k}{1 + r}$	\$210.29	\$460.98
Debt value	$D = L_r^* \cdot V_r^L$	\$783.09	\$858.29
Equity value	$E = C - G_r - D$	\$1,827.21	\$2,002.69
Beta of government	$\beta_r^G = \beta^U + (\beta^U - \beta^D) \cdot (r / (1 + r)) \cdot \tau \cdot D / G_r$	1.0138	1.0077
Beta of equity	$\beta_r^E = \beta^U + (\beta^U - \beta^D) \cdot (1 - r / (1 + r) \cdot \tau) \cdot L_r$	1.4181	1.4181
Gov. cost of capital	$k_r^G = k + (k - r) \cdot (r / (1 + r)) \cdot \tau \cdot D / G_r$	11.41%	11.41%
Equity cost of capital	$k_r^E = k + (k - r) \cdot (1 - r / (1 + r) \cdot \tau) \cdot L_r$	11.46%	11.46%
WACC	$WACC_r = k - r \cdot \tau \cdot L_r^* \cdot (1 + k) / (1 + r)$	9.194%	9.194%

Appendix 2 The Validity of the No Default Assumption

Table 1
The Fixed Debt Policy and the Validity of the No Default Assumption – A Simulation Analysis

This table presents the results of a simulation analysis where we examine if and when there is a default over a horizon of 500 years when assuming a fixed debt policy and different stochastic EBIT processes. The choice of 500 periods is arbitrary, however, as value contributions are decreasing over time a terminal value at period 500 has no substantial impact on the current value. Hence, the difference between a default-free firm and a firm, which may have a default after period 500, is not substantial. Altogether, we test four different processes. The results show that in general there does exist a possibility of default. However, there are also parameter constellations where a default does not occur over the horizon at least for three out of four considered process types. Column 1 contains the structure of the EBIT process. Column 2 shows the presumed distributions of the stochastic component. Column 3 reports the first period and column 4 shows the mean period of default while running 10,000 iterations per process. $N(a;b)$ denotes a normal distribution with mean a and standard deviation b , $U(a;b)$ denotes a uniform distribution over the interval $[a;b]$. For each simulation we assume the following initial values: $EBIT_0 = 44$ million US-dollars is the cross-sectional median of the firm-specific mean EBIT (COMPUSTAT item A178), calculated on all observations between 2000 and 2010 (after eliminating all negative values), $debt_0 = 114$ million US-dollars is the cross-sectional median of the firm-specific debt (sum of COMPUSTAT items A9 and A34) calculated on the same sample, whereas the risk-free rate $r = 0.05$ and growth rate $g = 0.03$ are in line with Nekrasov and Shroff (2009). We use the abbreviation EFM for the “European Financial Management”, JFE denotes the “Journal of Financial Economics”, SBR is “Schmalenbach Business Review” and TFR refers to “The Financial Review”.

EBIT-process	Assumptions	Default (earliest)	Default (mean period)
Arzac and Glosten, EFM 2005, p. 456:	$\tilde{\varepsilon}_t \sim N(0;0.01)$	No	-
$EBIT_t = EBIT_{t-1}(1+g)(1+\tilde{\varepsilon}_t)$	$\tilde{\varepsilon}_t \sim N(0;0.1)$	Yes (22)	233.8
	$\tilde{\varepsilon}_t \sim U(-0.175;0.175) \Rightarrow E(\tilde{\varepsilon}_t) = 0, \sigma(\tilde{\varepsilon}_t) \approx 0.1$	Yes (23)	234.1
Cooper and Nyborg, JFE 2006, p. 221:	$\tilde{\varepsilon}_t \sim N(0;10)$	Yes (4)	4
$EBIT_t = EBIT(1+g)^t + \tilde{\varepsilon}_t$	$\tilde{\varepsilon}_t \sim N(0;20)$	Yes (1)	5.5
	$\tilde{\varepsilon}_t \sim U(-35;35) \Rightarrow E(\tilde{\varepsilon}_t) = 0, \sigma(\tilde{\varepsilon}_t) \approx 20$	No	-
Giacotto, TFR 2007, pp. 259:	$\tilde{\varepsilon}_t \sim N(0;0.01)$	Yes (166)	212.89
$EBIT_t = EBIT_{t-1}(1+g_t)$ $g_t = \alpha\bar{g} + (1-\alpha)\tilde{g}_{t-1} + \tilde{\varepsilon}_t - \beta\tilde{\varepsilon}_{t-1}$ with $\alpha = 0.85$	$\tilde{\varepsilon}_t \sim N(0;0.005)$	Yes (187)	213.39
	$\tilde{\varepsilon}_t \sim U(-0.009;0.009) \Rightarrow E(\tilde{\varepsilon}_t) = 0, \sigma(\tilde{\varepsilon}_t) \approx 0.005$	Yes (189)	213.38
Richter, SBR 2001, p. 178:	$\tilde{g}_t \sim N(0.03;0.01)$	No	-
$EBIT_{t+1} = EBIT_t(1+\tilde{g}_{t+1}), E_t[g_{t+1}] > 0$	$\tilde{g}_t \sim N(0.03;0.1)$	Yes (23)	240.6
	$\tilde{g}_t \sim U(-0.145;0.205) \Rightarrow E(\tilde{g}_t) = 0.03 \sigma(\tilde{g}_t) \approx 0.1$	Yes (33)	238.9

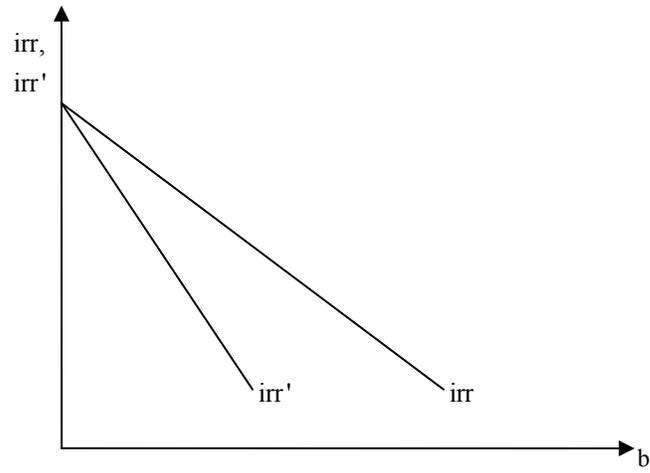


Fig. 1. The Investment Opportunity Set in a Steady State

This figure shows the investment opportunity set in a steady state. irr denotes the average internal rate of return and irr' is the marginal rate of return. b denotes the retention ratio.

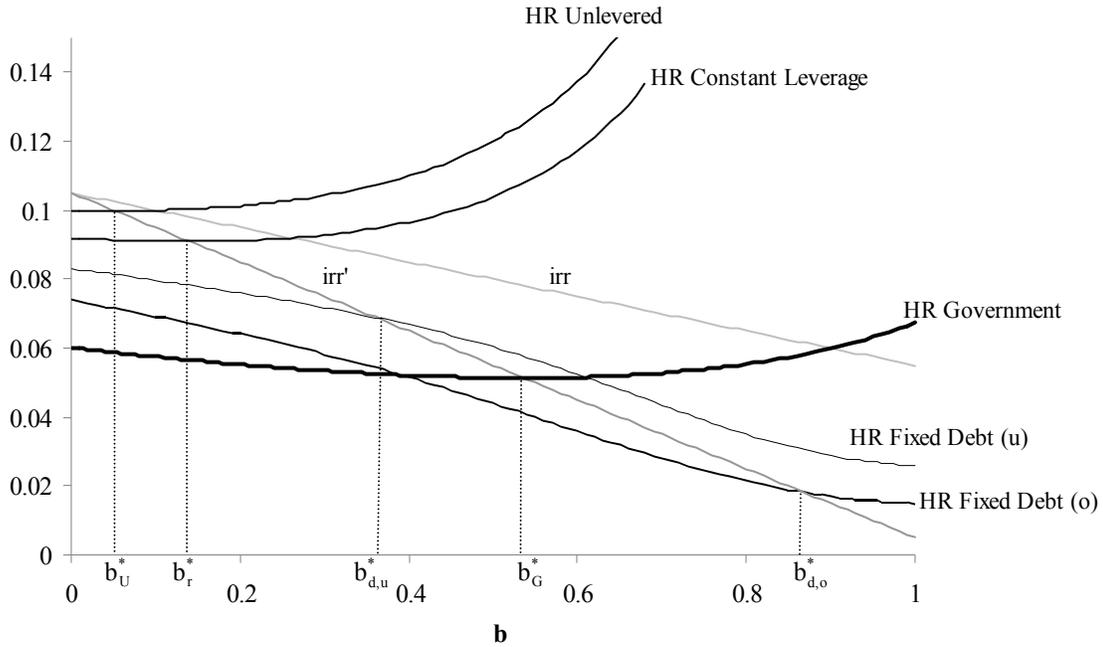


Fig. 2. Firm's and Government's Optimal Retention Ratios

This figure illustrates that a firm underinvests when it is unlevered or when it pursues a constant leverage policy. In contrast, when following a fixed debt policy the firm may under- or even overinvest. The intersection of the marginal rate of return $irr' = a - 2 \cdot q \cdot b$ with the corresponding HR functions each characterizes an optimal retention ratio. We assume $X = 400.00$, $\tau = 0.400$, $k = 0.100$, $r = 0.065$, $a = 0.105$ and $q = 0.050$. This parameter choice implies a government's optimal retention ratio $b_G^* = 0.536$. For the fixed debt case we choose a debt level $D = 1,400.00$ which is a sufficient level to encourage the firm to overinvest ($b_{d,o}^* = 0.873$). Alternatively, we assume a different debt level $D = 791.58$ that leads to underinvestment ($b_{d,u}^* = 0.362$) for a fixed debt policy. When pursuing a constant leverage policy, we also choose $D = 791.58$ and the firm underinvests ($b_r^* = 0.140$) even stronger in comparison to the fixed debt case. The lowest retention ratio is chosen if the firm is unlevered ($b_U^* = 0.051$). irr denotes the average internal rate of return according to equation (49) and irr' is the marginal the marginal rate of return as given by equation (50). HR Government denotes the hurdle rate function of the government according to equation (44), HR Constant Leverage (Fixed Debt) is the hurdle rate for the corresponding financial policy according to equations (41) and (42). u (o) denotes an underinvestment (overinvestment) scenario. b denotes the retention ratio.

Table 2
Overinvestment in the Case of a Fixed Debt Policy

This table shows parameter values that illustrate an overinvestment example in the case of a fixed debt policy. irr denotes the average internal rate of return and irr' is the marginal rate of return. g denotes the growth rate, V^U is the value of the unlevered company, G^U is the value of the unlevered tax payments and TS is the present value of tax shields. V_d^L denotes the enterprise value in the case of a fixed debt policy. C is the before tax value of the company. WL is the welfare loss due to the investment decision made by the shareholders. ΔG^U , ΔV^U and ΔV_d^L are the changes in the value of the unlevered tax payments, the unlevered firm value and the after-tax firm value induced by shareholders' overinvestment.

	$b_G^* = 0.536$	$b_d^* = 0.873$
irr	7.82%	6.13%
irr'	5.14%	1.77%
$\underline{g}(b_G^*)$	3.31%	
g	4.19%	5.36%
V^U	1,916.52	655.20
G^U	2,755.27	3,445.49
TS	1,577.77	3,182.53
V_d^L	3,494.29	3,837.73
C	4,671.79	4,100.69
ΔG^U		690.22
ΔV^U		-1,261.32
ΔV_d^L		343.44
$WL = -(\Delta V^U + \Delta G^U)$		571.10

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