Interbank Tiering and Money Center Banks

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Abstract

Interbank markets are tiered rather than flat, in the sense that many banks do not lend to each other directly but through money center banks acting as intermediaries. This paper captures the notion of tiering by designing a core-periphery model, and develops a procedure for fitting the model to a real-world network. Using Bundesbank data on bilateral interbank exposures among 1800 banks, we find strong evidence of tiering for the German banking system. Moreover, bank-specific features, such as balance sheet size, help explain how banks position themselves in the interbank market. This suggests that models with heterogenous banks could help shed light on how financial networks are formed.

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Introduction [to be revised]

This paper proposes the view that interbank markets can be tiered, operating in a hierarchical fashion whereby lower-tier banks deal with each other primarily through money center banks. It may seem unusual to focus on intermediation *between* banks; intermediation is traditionally regarded as the activity banks perform on behalf of *non-banks*, such as depositors and firms (Gurley and Shaw (1956), Diamond (1984)). The notion that banks build yet another layer of intermediation between themselves goes largely unnoticed in the banking literature. Yet hierarchical structures appear to characterise financial markets well beyond banking.

The interbank market is often modelled in the literature as a centralized exchange in which banks reconcile their deficits and surpluses (e.g., Bhattacharya and Gale (1987)). In reality, the interbank market is decentralized: deals are struck bilaterally between pairs of banks, not against a central counterparty (Stigum and Crescenzi (2007)). While some recent models recognize the bilateral nature of this market (e.g. Allen and Gale (2000), Freixas et al (2000), and Leitner (2005)), the presence of intermediaries, and hence its tiered character, has not been analyzed in any rigorous way. Yet it is by uncovering such structures that one can hope to make progress on understanding the role of banks in financial networks, and the meaning of macroprudential concepts like "too-connected-to-fail".

This paper defines interbank tiering and provides a network characterization founded on the economic concept of intermediation. The interbank market is tiered when some banks intermediate between other banks that do not extend credit (directly) among themselves. We capture this market structure by formulating a *core-periphery model* that refines the standard notion of intermediation. Core banks form a strict subset among bank intermediaries in that they both transact with each other and with other banks who transact only through core banks. To measure the extent of tiering in real-world networks, we propose a procedure, based on blockmodeling techniques, for fitting a model to empirical networks and implement it using a fast optimization algorithm. The procedure is shown to deliver a core which is a strict subset of intermediaries, excluding those that play no essential role in holding together the interbank market.

In defining tiering over interbank credit relations, we focus on a meaningful economic choice. In order to lend, a bank typically has to run creditworthiness checks (e.g. Broecker (1990)), the cost of which may limit the number of counterparties. A credit exposure is thus more likely to reflect an economic relationship than many other transactions, such as the submission of a payment. The payments literature uses the term tiering in related sense, to describe differential access to payment and settlement systems (CPSS (2003), Kahn and Roberds (2009)): in some systems, only few banks are direct members, and other banks have to transact through members to settle payments with each other (e.g. CHAPS in the United Kingdom).[1] While this automatically

[1] This literature focuses on the determinants of direct membership (Kahn and Roberds (2009), Galbiati and

gives rise to two tiers, it reflects the routing of payments (on behalf of customers) rather than the extension of credit between banks. Exposures, unlike payments, do not cease to exist once they have been made, and the structure of the resulting network is of greater relevance for financial stability.

In the empirical part, we rely on comprehensive Bundesbank statistics to construct the network of bilateral interbank positions between 1800 banks. The core comprises only 2.5% of interbank intermediaries, suggesting that tiering delivers a strong refinement of the concept of intermediation. Throughout the available time span (1999Q1– 2007Q4), the size and composition of the optimal core are stable. This supports the idea that we have identified a structural feature, one that has hitherto only been described in qualitative terms using aggregate data (Ehrmann and Worms (2004), Upper and Worms (2004)).

To test whether tiering provides a good description of the German banking system, the fit is compared to the distribution obtained from fitting 1000 networks formed by standard random processes. The German interbank network fits the core-periphery model eight times better than Erdös-Rényi random graphs and about two times better than scale-free networks of the same dimension and density. This amounts to strong evidence that the German banking system is tiered to an extent that cannot be replicated by random networks.

As tiering is not expected in random networks but arises from purposeful behavior, there must be economic reasons why the banking system organizes itself around a core of money center banks. The final part of the paper explores this direction, testing whether balance sheet variables predict the membership of a bank in the core. The probit regressions confirm that core banks tend to be large, which suggests that the presence of fixed costs, or economies of scale and scope, play a role in how banks choose to position themselves in networks.[2] We regard this link, between banking theory and network analysis, as a promising avenue for a better understanding of the formation of networks.

1 Tiering in the interbank market

This section first defines interbank tiering and provides a network characterization of the concept. It then develops a procedure for fitting the model to real-world networks and implements it through a fast algorithm. The concepts are illustrated by a simple example, and the procedure and hypothesis tests are applied to the large German interbank market in subsequent sections.

Giansante (2009)), involving legal or technological factors as much as economic considerations.

[2] Cases in which size and interconnectedness *deviate* are of interest too, especially at a time when regulators seek to substantiate the notions of 'too-big-to-fail' and 'too-connected-to-fail'.

1.1 From intermediation to tiering

Banks may rely on intermediaries for a variety of functions. One is liquidity distribution, the process of channeling funds from surplus banks to deficit banks (e.g. Niehans and Hewson (1976), Bruche and Suarez (2010)). Another is risk management: banks may place interbank deposits for purposes of diversification, risk-sharing and insurance (e.g. Allen and Gale (2000), Leitner (2005)). Banks may also take and place funds in different maturities to alter their maturity profile (e.g. Diamond (1991), Hellwig (1994)). For these and others functions (including custodian or settlement services), banks rely on intermediaries in ways that give rise to interbank credit exposures.

Definition 1: Interbank intermediation. *An interbank intermediary is a bank acting both as lender and borrower in the interbank market.*

This is the standard concept of financial intermediation, applied more narrowly to the banking market. The set of interbank intermediaries can be identified from existing banking data as the subset of banks recording both claims *and* liabilities vis-à-vis other banks on their balance sheet. Our concept of interbank tiering describes the interbank structure that arises when *some* banks intermediate between banks that *do not* transact among themselves.

Definition 2: Interbank tiering. *Some banks (the top tier) lend to each other, and intermediate between other banks who participate in the interbank market only via these top-tier banks.*

An interbank market is tiered when it is organized in layers, which we call tiers to evoke the hierarchical nature of the concept – in contrast with a "flat" structure without intermediaries. This can be expressed in terms of bilateral relations between top-tier and lower-tier banks:

$$\begin{cases} \text{1. Top-tier banks lend to each other,} \\ \text{2. lower-tier banks do not lend to each other,} \\ \text{3. top-tier bank lend to (some) lower-tier banks, and} \\ \text{4. top-tier banks borrow from (some) lower-tier banks.} \end{cases} \tag{1}$$

This formulation conveys several important points. Tiering is a structural property of the *system*, not a property of any individual bank. Furthermore, tiering is a *network* concept: the banks in the system are partitioned into two sets based on their *bilateral relations* with each other. At the same time, unlike other network concepts, it is founded on an *economic* concept that is central to banking and finance, intermediation. In fact, tiering is a *refinement* of intermediation: top-tier banks are special intermediaries that play a central role in holding together the interbank market.

Before developing a formal characterization, we provide a simple illustration of interbank tiering.

Example. Figure 1 (left panel) depicts a stylised interbank market comprised of 8 banks.[3] The arrows represent the direction of credit exposure, e.g. bank D lends to A. Banks $\{D, F, H\}$ are either lenders *or* borrowers, not both. The set of intermediaries thus consists of banks $\{A, B, C, E, G\}$. Bank C, for instance, intermediates from lender F to borrower H. It takes a chain of banks (involving A and C) to intermediate from D to H. The top tier consists of a strict subset of intermediaries, namely $\{A, B, C\}$ shown in solid color, while the remaining banks constitute the lower tier. For this partition of banks, the relations within and between the two sets exactly match the relations listed in (1). Banks E and G are intermediaries, but they belong to the lower tier because they are not sufficiently connected with other banks to qualify for the top tier (where they would violate the relations 1, 3 and 4). This reflects the fact that these two banks play no role in connecting lower-tier banks to the interbank market.

[Figure 1: Illustration of interbank tiering]

This example illustrates a perfectly tiered interbank structure. In reality, the presence of tiering will be a matter of degree. Much of what follows serves to develop methods that formalize how to think about the *distance* between real-world networks and perfectly tiered structures.

1.2 Network characterization of tiering

This section develops a structural representation for our definition of interbank tiering. This will serve as a benchmark model against which empirical interbank market structures can be assessed. A network consists of a set of nodes that are connected by links. Taking each bank as a node, the interbank positions between them constitute the network, which can be represented as a square matrix of dimension n equal to the number of banks in the system. The typical element (i, j) represents a gross interbank claim, the value of credit extended by bank i to bank j. Row i thus shows bank i's bilateral interbank claims, and column i shows the same bank's interbank liabilities to each of the banks in the system. The diagonal elements (i, i) are zero when treating banks as consolidated entities (with intragroup exposures netted out). Off-diagonal elements are positive, or zero in the absence of a bilateral position. Real-world interbank data typically give rise to directed, sparse and valued networks.[4] Since the concept of tiering is about the *structure* of linkages, we code the presence (or absence) of a link by 1

[3] The other panels of Figure 1 are discussed on page 9 where the example is continued.

[4] The networks are directed, because a claim of bank i on j (an asset of i) is not the same as a claim of j on i (a liability of i). They are sparse as only a small share of the $n(n-1)$ potential bilateral links are used at any point in time. Finally, interbank networks are valued because interbank positions are reported in monetary terms.

(or 0, respectively), as is common practice in network analysis. Therefore, the model, as well as the empirical interbank network in our application, will be non-symmetric binary matrices.

We now determine the shape of a network that captures a perfectly tiered structure. To do so, the bilateral relations (1) consistent with our definition of tiering are mapped into a matrix, M, with top-tier banks ordered first. For reasons that will become clear shortly, we shall call the set of top-tier banks "the core" (C), and the set of lower-tier banks "the periphery" (P). The nodes within each tier are *equivalent* as regards the nature of their linkages with other nodes. Hence it suffices to specify the generic relations within and between the two tiers in what is known as a *blockmodel*,[5]

$$M = \begin{pmatrix} CC & CP \\ PC & PP \end{pmatrix}.$$

The block denoted by CC ("core to core") specifies how top-tier banks relate to other core banks: when they all lend to each other, as specified in (1), CC is a block of ones (ignoring the zero diagonal). Likewise, periphery banks *not* lending to each other makes PP a square matrix of zeros. Core banks lending to some banks in the periphery means that CP contains at least one link in every row. Finally, core banks borrowing from some periphery banks makes PC a matrix with at least one 1 in every column. Our definition of tiering therefore translates into the choice and location of specific block types. (Other models would require other block types.) The blockmodel consists of a complete block (denoted **1**) and a zero block (**0**) on the diagonal which specifies relations *within* the tiers, and two off-diagonal blocks specifying relations *between* the tiers: CP must be row-regular, and PC column-regular.[6]

$$M = \begin{pmatrix} \mathbf{1} & \mathbf{RR} \\ \mathbf{CR} & \mathbf{0} \end{pmatrix}. \tag{2}$$

The size of M and its blocks need not be specified a priori; the number and identity of banks allocated to each tier will be determined endogenously. If c banks end up in the core, then the block CP, for instance, will be a matrix of dimension $c \times (n - c)$. One easily verifies that our simple example of tiering (Figure 1, left panel) conforms with the blockmodel M (with $n = 8$,

[5]Blockmodels are theoretical reductions of networks and have a long tradition in the analysis of social roles (Wasserman and Faust (1994)).

[6]These terms come from the literature on generalized blockmodeling (Doreian et al (2005)). A column-regular block, **CR**, has each column (but not necessarily each row) covered by at least one 1; the **RR** block has each row covered by at least one 1.

$c = 3$),

$$\begin{pmatrix} \mathbf{1} & \mathbf{RR} \\ \mathbf{CR} & \mathbf{0} \end{pmatrix} = \left(\begin{array}{ccc|ccccc} 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

Our network characterization of tiering is in fact a refinement of what is commonly called a *core-periphery model*. In social network analysis, this label is attached to any network with a dense cohesive core and a sparse periphery (Borgatti and Everett (1999)), as reflected in the diagonal blocks **1** and **0** in (2). However, the core-periphery model in the literature does *not* specify how the core and periphery are related to each other; the blocks on the off-diagonal could be of any type and are often ignored in the analysis (as recommended by Borgatti and Everett (1999)). In building on intermediation, our model of tiering *does* specify how the core and periphery should be related: core banks borrow from, and lend to, at least one bank in the periphery; they *intermediate* between banks in the periphery and thereby hold together the entire interbank market.

This particular focus on how the core and periphery are related is based on an economic rationale that seems appropriate for the interbank market. Core banks are in the market at all times and incur interbank positions with important counterparties in the normal course of business (hence $CC = \mathbf{1}$). Periphery banks, on the other hand, might only lend, or borrow, or might not participate in the interbank market at all when they have no deficits or risks to cover at that moment. It would be too restrictive to impose that *every* bank in the periphery has to be connected;[7] but the periphery *as a whole* should certainly be linked to the core, else there is not really a single interbank market to speak of.[8] The right balance is found by placing strong restrictions only on core banks: every core bank must be connected to some bank(s) in the periphery, but the converse need not hold. This is what the choice of row- and column-regular blocks on the off-diagonal of M accomplishes. Moreover, these block types also

[7] This would be the result of defining CP and PC as complete (**1**) or regular blocks. A regular block has at least one 1 in every row *and* column, implying that every periphery bank lends to, *and* borrows from, some bank in the core (which would make all banks in the system intermediaries).

[8] This degenerate case of an unconnected periphery is permitted in the weak core-periphery model (with CP and PC zero blocks) discussed by Borgatti and Everett (1999)).

preserve flexibility in specifying only the minimum requirement; in applications, this allows for the greatest number of matrices to be consistent with this requirement and thereby helps the model adapt to a variety of empirical networks.

1.3 Testing for structure

We now focus on the question of how to determine the extent to which an observed real-world network exhibits tiering. But how can one test for the *structure* of a network? Visual inspection is instructive but inconclusive for large networks, and traditional network statistics do not relate to any underlying model, tiered or otherwise. Our approach is to compare the network of interest with the model in terms of a measure of distance that aggregates the structural inconsistencies between them. If the observed network and the best-fitting tiering model remain at great distance from each other, then the network does not have a tiered structure.

We first formulate a statistical procedure for fitting the model M to an observed network N. This can be thought of as running a regression, but instead of estimating the parameter β that achieves the best linear fit, one determines the optimal set of core banks that achieves the best structural match between N and M. We show that the solution has the desirable property that the core is a strict subset of all intermediaries. Finding this solution is a large-scale problem in combinatorial optimization for which we develop a fast algorithm. We then evaluate the degree of tiering in the observed network by testing the goodness of fit against the distribution obtained from fitting random networks for which tiering is not expected to emerge.

Fitting the model to a network

The tiering model M serves as the benchmark for assessing the extent of tiering inherent in an observed interbank network N. These two objects have to be made comparable. The observed network N is a square matrix of dimension n equal to the number of banks, with $N_{ij} = 1$ if bank i lends to bank $j \neq i$, and $N_{ij} = 0$ otherwise. The model M, on the other hand, is a generic structure that embodies the relations (1) for any dimension. The fitting procedure involves the two steps of (a) defining a measure of distance between the network and the model M of the same dimension, using (2) as the matching criterion; and (b) solving for the optimal (distance-minimizing) partition of banks into core and periphery. Working with the optimal fit takes care of the problem that tiering is a *qualitative* concept that does not depend on the exact size of the core (or periphery) as long as there are two tiers.

The measure of distance we adopt, following the generalized blockmodeling approach of Doreian et al (2005), is a total error score. It aggregates the number of inconsistencies between the observed network and the chosen model. Consider an arbitrary partition where c banks are considered for the core, leaving $(n-c)$ banks in the periphery. Denote the set of core banks by C; ordering core banks first (and rearranging N by permutation accordingly) makes $C = \{1, 2, ..., c\}$. This partition divides the observed matrix N into four blocks, and the model M predicts how each block *should* look like in a perfectly tiered network of the same dimension. In particular, the top tier CC should be a complete block $\mathbf{1}$ of size c^2, so any missing link (outside the diagonal) presents an inconsistency with the model (2), as one core bank has no exposure to another. Likewise, any observed link within the periphery (PP) constitutes an error relative to M, as periphery banks should not transact directly among each other in a perfectly tiered market. Errors in the off-diagonal blocks penalize zero rows (columns), because these are inconsistent with row-regularity (column-regularity, respectively): a zero row in CP indicates that a core bank fails to lend to *any* of the $(n-c)$ banks in the periphery, violating a defining feature of core banks. Similarly, a zero column in PC shows that the corresponding core bank does not borrow at all from the periphery, producing as many errors as there are banks in the periphery $(n-c)$. The aggregate errors in each of these blocks are thus given by the following sums,

$$E = \begin{pmatrix} c\,(c-1) - \sum_{i \in C} \sum_{j \in C} N_{ij} & (n-c) \sum_{i \in C} \max \left\{0, 1 - \sum_{j \notin C} N_{ij}\right\} \\ (n-c) \sum_{j \in C} \max \left\{0, 1 - \sum_{i \notin C} N_{ij}\right\} & \sum_{i \notin C} \sum_{j \notin C} N_{ij} \end{pmatrix}. \quad (3)$$

The total error score aggregates the errors across the four blocks.[9] We normalize the error score by the total number of links in the observed network,

$$e = \frac{E_{11} + E_{22} + (E_{12} + E_{21})}{\sum_i \sum_j N_{ij}}. \quad (4)$$

The total error score is our measure of distance; it is a function since every possible partition into two tiers is associated with a particular value of e. Denote this function by $e(C)$, where C stands for the set of banks under consideration for the core. The optimal core, C^*, is the set(s) of banks that produce the smallest distance to the model M of the same dimension,

$$\begin{aligned} C^* &= \arg\min \; e(C) \\ &= \{C \in \Gamma \mid e(C) \leq e(c) \; \forall c \in \Gamma\}, \end{aligned} \quad (5)$$

where Γ denotes all strict and non-empty subsets of the population $\{1, 2, ..., n\}$. Intuitively, the expression (5) determines the number and identity of banks in N that are core banks in the sense of the interbank tiering model. The following example illustrates in the simplest way

[9] The aggregation of errors can be adapted to cases in which one type of error is more consequential than another. E.g. multiplying $(E_{12} + E_{21})$ by a parameter below unity deemphasises the relation between core and periphery; multiplying E_{11} by a number above unity will yield a solution with a smaller tightly connected core. As no theoretical priors on intermediation suggest otherwise, we use the equally weighted aggregation of errors, in line with the overall dimension of the network.

how structural inconsistencies between N and M are measured by the distance function, and minimized by the optimal core.

Example continued. Reconsider Figure 1, where the left panel shows our earlier example of a tiered structure (M). The other panels depict examples of networks that are not perfectly tiered (N). In the middle panel, suppose we knew that banks $\{A, B, C\}$ are good candidates for the core. If so, however, we observe that one core bank (B) does not lend to another core bank C, and periphery bank D lends directly to another (H). Accordingly, the matrix (3) yields one error in each of the diagonal blocks CC and PP. As no other partition attains a lower error score, $\{A, B, C\}$ remains the optimal core that minimizes the total error score to $e(C^*) = 2/13$.

Suppose we conjecture that $\{A, B, C\}$ also forms the core of the network in the right panel. We observe that one putative core bank does not lend to the periphery at all; this immediately generates 5 fitting errors in block CP for C's failure to lend to *any* of the 5 banks in the periphery. Moving C to the periphery instead causes a single error (its continued link with periphery bank F), in addition to the existing error (D lending to H). The distance between network and model can thus be reduced by placing bank C in the periphery, i.e. by considering a tiering model with only two nodes in the core (and six in the periphery). The optimal fit yields two errors in the (enlarged) periphery, none in the (reduced) core $\{A, B\}$, and none again in the off-diagonal blocks, or a total score of $e(C^*) = 2/12$. The new core excludes bank C which obviously remains an intermediary, illustrating that the core comprises only those intermediaries that intermediate between banks in the periphery, as required by Definition 2.

Real-world network are far more complex than this example suggests, with structures that may be arbitrarily far removed from that of a tiered market. This makes it essential to understand the properties of the optimal fit and to develop an efficient procedure for arriving at this solution. We now show that the solution preserves the main features illustrated in this simple example.

Properties of the solution

The procedure of minimizing the distance between model M and network N delivers the optimal partition of banks into core and periphery. Based on our definition of distance (3)-(4), the solution must have the following desirable properties:

Proposition 1:

(a) The presence of intermediaries is necessary and sufficient for a core-periphery structure:
  (i) A network without intermediaries has no core.
  (ii) A network with intermediaries has a core (and a periphery under one weak condition).

(b) The core is a (strict) subset of the set of intermediaries:
(i) All core banks are intermediaries, but
(ii) Intermediaries are not part of the core if they do not lend to, *or* do not borrow from, the periphery.

Proof: see Appendix A. The first property relates to existence, and shows that the distance-minimizing procedure can identify a core-periphery structure in virtually all networks. The sufficient condition for a core is the presence of at least one intermediary. A periphery always exists under the weak (and sufficient) condition that the network contains either unattached banks, or one missing bilateral link. This is intuitive, since an interbank market in which *every* bank lends to all other banks, as in Allen and Gale (2000), cannot be regarded as tiered but rather as "flat," since banks are all equal in their connection patterns. The core-periphery model can thus be fitted under conditions that are satisfied by all realistic interbank networks.

The second part shows that our concept of tiering delivers a useful *refinement* of the concept of intermediation. The core is a strict subset of all intermediaries, as illustrated in Figure 2. Core banks are *special* intermediaries that intermediate between banks in the periphery. While this property is, of course, in line with our definition of tiering (and thus embodied in M), the result states that this property carries over exactly to the solution when fitting M to an observed network N. This is remarkable, because one would expect any statistical fitting procedure on a large network to produce some errors in every block of (3). Here, however, the off-diagonal blocks governing the relations *between* core and periphery have error scores of exactly zero. Consequently, the error score (4) at the optimum takes the simple form

$$e(C^*) = \frac{E_{11} + E_{22}}{\sum_i \sum_j N_{ij}}. \tag{6}$$

We have encountered these properties of the solution in the example above, where off-diagonal errors were zero and the optimal core $\{A, B\}$ was a strict subset of all intermediaries $\{A, B, C, E, G\}$. The traditional core-periphery model, which disregards off-diagonal blocks (Borgatti and Everett (1999)), would have retained bank C in the core, even though this bank no longer intermediates between banks in the periphery.

[Figure 2: Tiering as a refinement of intermediation]

Implementation

Fitting the model to a real-world network is a large-scale problem in combinatorial optimization. Only for very small networks can the solution be found by exhaustive search. In our example with 8 banks (page 9), for instance, computing the total error scores for each of the $2^8 = 256$ possible partitions confirms that $\{A, B\}$ is indeed the (unique) solution that minimizes the error function. This brute-force approach becomes infeasible for larger networks. A medium-sized banking system of some 250 banks already requires on the order of 10^{78} possible subsets (2^n) to be evaluated for determining the optimal core. The problem of finding an optimal subset – which our paper shares with Kirman et al (2007) and Ballester et al (2009) – is *NP-hard*. The computational complexity of such problems rises *exponentially* with n, so that they cannot be solved by exhaustive search. The goal of fitting the model to realistic networks, such as the German interbank market with close to 2'000 active banks, calls for a more efficient procedure.

Our implementation thus relies on a sequential optimization algorithm, which follows closely the switching logic employed in our proof of Proposition 1. An initial random partition is evaluated and improved upon by moving banks between core and periphery until the total error score (4) can no longer be reduced. The *greedy* version of our algorithm follows the steepest descent, switching from one tier to another the bank that contributes most to the error score at each iteration. To avoid running into local optima, a second algorithm employs *simulated annealing*, which allows for a degree of randomness when moving banks that declines monotonically as the optimum is being approached. One way to test whether the procedure returns a global optimum is by inspecting the associated E, since we know from Proposition 1 that a genuine solution necessarily comes with a diagonal error matrix. Appendix B describes further robustness checks we performed to ascertain that the procedure converges on a global optimum. The main programming challenge consisted of reducing the algorithm's polynomial running time from order n^3 to n^1. This made the algorithm sufficiently fast for the repeated applications necessary for hypothesis testing.

Hypothesis test against random networks

Having shown how to fit the model, we must address the issue of significance: how can one evaluate the extent to which the observed network exhibits tiering? The closer the network resembles a tiered structure, the lower will be the error score (6). For a formal test, one must compare the distance between network and model to some benchmark. Selecting a benchmark, however, is not straightforward since we are assessing a qualitative structural feature. Moreover, it would be questionable – as in econometrics – to change, without a theoretical basis, the underlying model only to improve the statistical fit. It is easy to reduce the total error score by choice of a weaker model, for instance by replacing the complete block **1** in (2) by a (more accommodative) regular block.[10] Doing so would be unprincipled and undermine the theoretical arguments advanced in Section 1.2 that led to this particular model. We therefore adopt a

[10] Model selection remains an under-explored area in blockmodeling. Doreian et al (2005) provide no clear guidance, although they rightly caution against selecting among block types to minimize the number of structural inconsistencies.

different strategy for evaluating significance.[11]

In a first step, we assess whether a tiering model is worth fitting at all. Recall that our measure of distance (4)-(6) normalizes the aggregate error by the total number of links in the observed network, $\Sigma\Sigma N_{ij}$. This is also the maximum error under the alternative hypothesis that the network comprises only a periphery. The minimum distance $e(C^*)$ can therefore be used in a basic test, similar in spirit to an F-test of joint significance on whether it is worth including regressors at all.[12] If $e(C^*) \geq 1$, then there is no value in fitting a tiering model: doing so generates more structural inconsistencies than does a "flat" model with a periphery alone. In that case there is no evidence of a core standing out as a separate tier.[13] We require that $e(C^*)$ attains a value well below unity to proceed.

In the second step, our strategy is to vary the data rather than the model: we test the total error score against the Monte Carlo distribution function from a data-generating process in which tiering is not expected to emerge. In particular, the error $e(C^*)$ associated with the observed network N is tested against the error distribution obtained by fitting simulated networks where links are formed by exogenous statistical processes. The most popular classes are *random graphs* introduced by Erdös and Rényi, and *scale-free networks* popularised by Albert and Barabási and widely observed in the natural sciences (Newman et al (2006)):

- A *random graph* is obtained by connecting any two nodes with a fixed and independent probability p. Any realisation of such a network also has an expected density of p. A node can be expected to have a *degree*, or number of links, of $p(n-1)$ – on each side in the case of a directed network. The expected degree distribution around this characteristic value is Binomial, converging to Poisson for large n.
- A *scale-free network*, on the other hand, has no characteristic scale: nodes with lower degree are proportionately more likely than nodes with k times that degree, for any k. The degree distribution thus follows a power law. One statistical process giving rise to scale-free networks is known as *preferential attachment*, whereby new nodes attach to existing nodes with a probability proportional to the latters' degrees. This formation process tends to produce a few highly connected hubs, suggesting that scale-free networks match interbank networks more closely than do random networks.

[11] Our approach of comparing a network to a *specific model* contrasts with the powerful maximum likelihood method applied by Copic et al (2009), which finds the partition with the highest probability of producing the observed network. Rather than fitting an underlying model, their method thus *specifies* the likeliest community structure, defined as groups of nodes more likely to connect within than across groups. (Their notion of community structure is not directly applicable in our context, e.g. links within the periphery are *less* likely).

[12] This test requires no distribution, since the observed network comprises the full population (not only a sample) of nodes.

[13] The other side of the test (a "flat" model with only a core) can be disregarded, except in the unusual case where the density of the observed network exceeds 50%.

Random and scale-free models are not hierarchical in nature (Ravasz and Barabási (2003)). The purely statistical nature of these network formation processes is at odds with the idea that banks, by purposeful economic choice, organize themselves around a core of intermediaries, giving rise to interbank tiering. We therefore generate 1000 random networks of the same dimension and density as the observed network N, and fit the model M to *every* realization. This allows to trace out an empirical distribution function F_e for the error score in an environment where tiering occurs only by chance. We say that N exhibits a significant degree of tiering if the associated test statistic $e(C^*)$ is closer to zero than the bottom percentile of the distribution function found for random networks,

$$\text{Reject H}_0 \text{ if:} \quad e(C^*) < F_e(0.01).$$

This significance test can be conducted separately for each class of random networks, Erdös-Rényi and scale-free. It can also be understood as rejecting the hypothesis that networks formed by random processes would produce the extent of tiering observed in N. As tiering is not expected to arise in such networks, it must be the result of incentives of banks for linking to each other in this particular way. Following our application, we explore this direction further in section 3.

2 Application to the German banking system

2.1 Constructing the interbank network

We employ a set of comprehensive banking statistics known as the *"Gross- und Millionenkredit-statistik"* (statistics on large loans and concentrated exposures). The data are compiled by the Evidenzzentrale der Deutschen Bundesbank. According to the Banking Act of 1998, financial institutions located in Germany are mandated to report on a quarterly basis each counterparty to whom they have extended credit in the amount of more than €1.5 million or 10% of their liable capital. If either threshold is exceeded at any time during the quarter, the lender reports outstanding claims (both long-term and short-term) as they stand at the end of the quarter. From these reports, the Bundesbank assembles the central credit register, which is employed by reporting institutions for monitoring borrower indebtedness and by the authorities for monitoring the financial system.

The nature of these data presents several advantages. Claims are reported with a full counterparty breakdown vis-à-vis thousands of banks and firms. The bilateral positions are therefore directly observed and need not be estimated as in many other studies.[14] This makes it legiti-

[14] In many existing interbank market studies, bilateral positions have to be either reconstructed from payment

mate to apply network methods. Second, positions are quoted in monetary values (in millions of euros), indicating both the presence and strength of bilateral links. As the concept of tiering is about the *structure* of linkages, however, the monetary values are used here only to indicate the presence of a credit exposure. Third, the data are available on a quarterly basis since 1999Q1, which allows us to observe the structure of the network over time.

We gathered the reported bilateral positions between 2200 banks to construct the interbank network. To capture relations between legal entities (rather than internal markets), we consolidated reporting banks by ownership at the level of the Konzern (bank holding company) and thereby purged the data of intragroup positions. We also excluded cross-border linkages in order obtain a self-contained network, since any further linkages booked by counterparties abroad remain unobserved. The network therefore contains the bilateral interbank exposures between some 2000 banks (including foreign banks) located in Germany.

The basic statistics convey a first impression. The German banking system is one of the four largest in the world, with asset size of €7.6 trillion ($11 trillion) at end-2007. Reflecting the key role of the interbank market, interbank positions sum to €X.X trillion, making up a sizeable share of banks' balance sheets. Even after Konzern-level consolidation, the number of active banks in the interbank market varies between 1760 to 1802 for our sample period. This set comprises, on average, 40 private credit banks (Kreditbanken), 400 savings banks (Sparkassen), 1150 credit unions (Kreditgenossenschaften), and 200 special purpose banks. Yet the network is sparse, with a density on the order of 0.41% of possible links (0.61% when excluding inactive banks).[15] This sparsity suggests the presence of a discernible structure. The German banking system thus represents a network of interest not only in its own right, but also affords an opportunity to test whether a network of this size and intricacy can be characterized with a simple core-periphery structure.

2.2 Fitting the core-periphery model

We now fit the tiered structure M to the German interbank network. The first results focus on a single representative quarter, 2003Q2, in which 1802 banks participated in the interbank market, 1671 as intermediaries, 67 as lenders only, and 64 as borrowers only. Using the procedure developed in Section 1.3, the optimal core was found to include 45 banks.[16] This is indeed a strict subset, comprising only 2.7% of intermediaries. As expected from Proposition 1, the core

flows (e.g. Furfine (2003), and Bech and Atalay (2008)), or estimated from balance sheet data using entropy methods (Upper and Worms (2004)). Mistrulli (2007) documents the resulting bias when estimating contagion. More importantly for our purposes, the entropy method spreads linkages so evenly that essential qualitative features of the network structure will disappear.

[15] Further network measures for the German interbank market are reported in Craig, Fecht, and von Borstel (2009).

[16] The optimal fit was robust across algorithms, as described in Appendix B.

includes only intermediaries that borrow from, and lend to, the periphery (the lower tier). The core excludes all those banks that appear as intermediaries in the data but play no essential role in the network: they only transform their own maturity profile by taking and placing funds in different maturities, often with a single counterparty in the core (see also Ehrmann and Worms (2004)).

This confirms that the core is a strong refinement of the concept of intermediation. The core here is much smaller than what is sometimes called core in other network studies.[17] In building on intermediation, our model of tiering leads to a tighter core, comprising only 1.9% of all banks in the network; yet, without this core the interbank market would not be a single market.

Total error score (4) of the optimal fit came to 0.122, or 12.2% of network links. This is an average of 1.3 errors per bank, compared to the average 11 active links per bank. When normalized by the number of cells (potential links) in the network, only 0.074% of cells in the network prove inconsistent with model M. The total number of errors reached its minimum at 2406, comprising 683 errors (missing interbank links) within the core. Still, the density of the core is 66%, more than 100 times greater than the average density of the full network. The error matrix inevitably featured no errors in the off-diagonal blocks, consistent with the theoretical properties. The majority of errors (1723) therefore occurred on account of direct lending within the periphery.

[**Figure 3: Structural stability over time**]

We track the evolution of the network on a quarterly basis from 1999Q1 through 2007Q4. The structure we identified is highly persistent. First, the *size* of the core and associated error score are stable over time (see Figure 3). The exception is the apparent break in series in 2006Q3, where a number of mergers reduced the size from 44-46 banks prior to this date, to 35-37 banks thereafter.[18] Importantly, the *composition* of banks within the core also remains remarkably

[17] For Broder et al (2000), the core of the worldwide web is the *giant strongly connected component* (GSCC), the set of pages that can reach one another through hyperlinks in both directions. Pages that can reach (or can be reached by) the core make up the *giant in-component* (or *out-component*, respectively). Broder et al (2000) and subsequent studies thus use the core-periphery notion in a weaker sense of "reachability", regardless of how many links (and thus intermediaries) it takes for one page to reach an other. As a result, their core is a large subset (28%) of all pages in the sample. Applied to the Fedwire payment network, Soramäki et al (2007) find the GSCC to comprise 66% of banks in the network.

[18] A number of mergers among banks in the core occurred, so the new core became a subset of the old core including the consolidated banks.

stable over time. This can be shown by means of the estimated transition matrix,

$$P(s'|s) = \begin{pmatrix} 0.940 & 0.049 & 0.011 \\ 0.001 & 0.991 & 0.008 \\ 0 & 0 & 1 \end{pmatrix}.$$

The element P_{12} represents the frequency with which core banks (state $s = 1$) move to the periphery (state $s' = 2$) over time. The third state (outside the sample) takes care of entry to, and exit from, the banking population. The fact that the values on the diagonal are close to unity confirms that banks tend to remain in the same tier (core or periphery). Estimating a separate transition matrix for each quarter demonstrates its stability over time (Figure 4).[19]

[**Figure 4: Transition probabilities over time**]

These findings support the idea that we have identified a truly structural feature of the interbank market. The persistence of this tiered structure poses a challenge to interbank models building on Diamond and Dybvig (1983). If unexpected liquidity shocks were the basis for interbank activity, should the observed linkages not be as random as the shocks? If so, the observed network should change unpredictably every period. It is the presence of a persistent structure that makes empirical interbank contagion exercises meaningful.

Robustness

Before evaluating the statistical significance of tiering, it is important to address potential caveats. One concern relates to the way the banking statistics are collected: could the reporting threshold (€1.5 million or 10% of their liable capital) bias the results? To test this possibility, we performed a censoring test whereby the model was fitted to networks defined by successively higher thresholds (from €1.5 to 100 millions, where only 50% of the value of reported positions remained in the network). The tiered structure remained unaffected, and the error score declined with each iteration. Apparently, much of the direct lending within the periphery is in smaller denominations that dropped out as the censoring threshold increased. Indeed, the value of lending within the periphery accounts for less than 2% of total interbank credit. This logic in reverse suggests that, if the reporting threshold were zero, one would still observe a tiered structure, though with more direct lending within in the periphery.

A more important question is whether legal structure and public ownership determines all

[19] The said merger activity among core banks makes the first row of P become $\begin{pmatrix} 0.63 & 0.22 & 0.15 \end{pmatrix}$ for the single quarter 2006Q3.

the network properties of the German banking system. The (public) savings banks have a special relationship with their respective public *Landesbanken*, which provide borrowing and lending services to them. In the same way, credit union banks have a special relationship with their central cooperative banks. These pillars or silos, and the tiering within them, are widely noted features of the German banking system, and are discussed in the interbank context by Ehrmann and Worms (2004), and Upper and Worms (2004). However, the observed network is not simply an institutional artifact but is driven by economic choices. Banks are free to lend and borrow from other banks throughout the entire system, and are not restricted to dealing with the head institution of their pillar. The data indeed shows direct linkages between periphery banks across different pillars. Moreover, the tiered network structure we identified predates subsequent institutional developments: Guinnane (2002) describes how the regional head institutions arose, in the 19th century, to provide much-needed intermediation services to the regionally dispersed credit unions and savings banks, predating major ownership changes and legal developments during the 20th century.

The view that economic motives, not only institutional factors, give rise to a core-periphery structure, can also be examined by removing various segments, or their respective head institutions, from the network (Figure 5). First, the two most connected banks (legal head institutions) were removed from the network along with all of their links. These two banks together maintain so many links that their number exceeds the total links of the next fifteen banks and so could greatly affect the error score. The estimated core of the reduced network reveals a time series of cores with essentially the same properties and banks as the original network. Other configurations of deleted banks yielded similar results.

The most drastic experiment was the entire removal of the two pillars most likely to be shaped by legal factors, the savings banks and credit cooperatives. This was to test whether tiering would occur in the remaining – and least regulated – segment of the German banking system. Once again, the presence of a core remains a consistent feature, varying fairly smoothly between 22 and 27 during the 36 quarters (Figure 5, solid lines). This is in spite of considerable merger activity in this segment of the banking industry over the sample period.[20]

[**Figure 5: Robustness checks**]

A more general concern could be that the model is not sufficiently sophisticated to capture the structure of the German (or any other) banking system. Our preference for the simple core-periphery model M is that it builds on intermediation. The procedure for fitting networks, however, allows for more general models which permits to match additional features of a given network. One can adapt the model to the vertical pillar structure of the German banking

[20]Interestingly, the structural break in 2006Q3 for the entire bank population is now absent; this is further indication that it occurred within the cooperative and savings bank sectors.

system, for instance, by replacing the row- and column-regular blocks in (2) by row- and column-*functional* blocks.[21] To extend the model to three tiers, the blockmodel is expanded to include a semi-periphery. Doing so for the German system would help distinguish regional intermediaries from the (few) genuine core banks intermediating aross the entire country.[22]

Significance

The core-periphery structure appears robust and stable over time, but is the fit sufficiently tight to conclude that the interbank market is genuinely tiered? The screening test described in Section 1.3 is easily passed: $e(C^*) = 0.122$ falls well below unity. This small a distance between network and model demonstrates that the tiered structure is a much better benchmark than the alternative comprising only a periphery.

In the second step, we test this score against the error distributions from fitting random networks. We generate 1000 Erdös-Rényi random graphs and 1000 scale-free networks of the same dimension and density as the German interbank network ($n = 1802; d = 0.61\%$). We then fitted M to each realization, and traced out the distributions F_e against which to assess the error score of the German network. Figure 6 shows the histograms of the normalized error scores (4) for each class of random networks separately.[23]

The error score distributions show that both classes of random networks exhibit fairly tight statistical properties.[24] The Erdös-Rényi random graphs show error scores highly concentrated around 0.983. This is so close to unity that there really no value in identifying a core of random graphs. Importantly, even the best-fitting realization produced an error score of 0.981, more than 8 times that of the German interbank network.

[**Figure 6: German Result against Error Score Density**]

Clearly, the distance from the German interbank network to the tiering model is orders of magnitude smaller than that from fitting random networks. This amounts to strong evidence of tiering for the German banking system, to an extent that cannot be replicated by random networks.

[21] A row-functional block (Doreian et al (2005)) in our context implies that any bank in the periphery relates to a *single* bank in the core.

[22] One indication in favour of three tiers is provided by the simple experiment of fitting the tiering model once more, only on the network among 45 core banks. This delivers an "inner core" of 28 banks with an error of 221 (17% of links).

[23] See Appendix B on the robustness checks we used to ascertain that the test distributions reflect the intrinsic randomness of networks, rather than stochastic output from an unreliable procedure.

[24] Scale-free networks consistently produced cores of size 55-57. Random graphs featured cores of size 17 or 18, in 86% and 14% of cases, respectively.

The scale-free network comes much closer.[25] This was to be expected, since scale-free networks are known to produce well-connected hubs that characterize many real networks, including interbank markets (Boss et al (2004)). Even so, none of the 1000 realizations of scale-free networks produced an error score of less than 0.204, a distance larger by a factor of 1.8 than that of the German network.

The goodness of fit for the German interbank network thus lies outside any conceivable percentile of the error distribution for both classes of random networks. We can therefore reject the hypothesis that networks formed randomly produce the extent of tiering evidenced by the German banking system. The core-periphery model is a much closer description of the German interbank network than of networks formed by statistical mechanics. We conclude that tiering observed in Germany is not likely to be the result of a random process of network formation. Indeed, the statistical approach to network formation is ill-suited for social and economic networks which are the result of purposeful activity by agents weighing the cost and benefits of forming links (Goyal (2007), Jackson (2008)). This is the direction to which we turn next.

3 Interbank tiering and money center banks [TO BE REVISED]

Tiering captures a structural feature of the the interbank market and allocates banks into core and periphery. As is typical for network analysis, this allocation is derived from the pattern of linkages alone: network statistics are calculated disregarding the attributes of individual nodes, such as bank size. In this section, we begin to explore whether individual bank features help explain how banks position themselves in the interbank network. We regard this link as a bridge between banking theory and network analysis, essential for a better understanding of the formation of interbank networks.

3.1 What makes a money center bank?

A bank in the core of a tiered interbank market can be regarded as a *money center bank*. This term is generally associated with large banks dominating wholesale activity in money markets. In addition to running traditional banking operations, money center banks provide clearing and correspondent banking services, and act as dealers in a broad range of markets, including government securities, FX, derivatives and offshore markets (Crescenzi and Stigum (2007) chapter 6). As such, money center banks are likely to be those intermediaries occupying

[25] Interestingly, the Monte Carlo experiments produced binning into four distinct error score classes (red in Figure 6). We made considerable efforts to ensure that these were not local minima, especially for the clusters around higher error scores. Clearly, more work needs to be done to uncover the reasons behind this phenomenon.

the special network position we identify as the core. In this network sense, money center banks play a central role among banks, in dealing among themselves and tying in the periphery.

In this section we ask: what kind of banks make it to the core? For the 1800 active banks in the German interbank network, we obtained balance sheet variables from the banking data collected by the Bundesbank's statistics department (*"monatliche Bilanzstatistik"*).[26] The idea is to test whether the membership of a bank in the core can be predicted by banks' own attributes. We specify a probit regression, where the dependent variable b is banks' network positions, with $b_i = 1$ if bank i was found to be part of the core in section 2, and $b_i = 0$ otherwise,

$$prob(b_i = 1) = \Phi(x_i'\beta).$$

The first column of Table 1 reports the simplest regression with bank size as the sole explanatory variable. The log of total bank assets (lassets) is highly significant, with a coefficient of 0.77. Indeed, size is a fairly reliable classifier. The average size of banks in the core is 51 times that of banks in the periphery; for equity, the factor is 54. Hence the banks in the core tend to be large, while smaller banks are located in the periphery of the interbank network. This result is in line with priors on money center banks, and with earlier studies on interbank markets. For example, the federal funds market is known to have natural lenders and borrowers, with small banks turning over surplus funds to large banks with extensive investment opportunities (Allen and Saunders (1986)). Further back in US monetary history, a related phenomenon known as reserve pyramiding occurred at seasonal frequency (White (1983)).

There are, however, several outliers that raise intriguing issues. Several very large banks were found to be far less connected than were other core banks – those banks might have focussed their business on capital markets and international activity to a greater extent. From the perspective of financial stability, it remains an open question whether "too-big-to-fail" or "too-connected-to-fail" is the more relevant criterion.

Following Craig, Fecht, and Von Borstel (2009) and Upper and Worms (2002), we gauge a bank's systemic relevance by measuring the damage its failure inflicts upon the rest of the system. Such simulations often require a loss given default (LGD) which often is not known, although Upper and Worms try several. Craig, Fecht, and Von Borstel assess the systemic relevance of a particular bank by measuring the LGD that is needed before the bank failure cause a systemic crisis (defined as 25% of system assets in default, given that LGD.) Because such an LGD goes down as the systemic relevance of a bank goes up, we define a variable "systemic" which is the inverse of the minimum LGD needed to cause a systemic crisis. The second column of Table 1 reports results of the probit of core membership as a function of the variable systemic. As is clear, systemic relevance and core membership are very highly

[26] The statistics on bank capital were separately obtained from the Bundesbank's Supervision Department (Banken- und Finanzaufsicht).

correlated. The third column of Table 1 utilizes a network concept, that of "betweeness" to examine whether the network position of the bank is correlated with whether it is in the core. These results make clear that the network position of the bank is also highly correlated with membership in the core.

Having said this, one could argue that those banks that are central in a network are also banks that are larger banks in terms of assets, and are also banks which are systemically more important. Indeed one could argue that the first three columns of Table 1 all make the same point. One might object that bank size is not exogenous with respect to network position. The intermediation function that money center banks perform, by borrowing and lending in the interbank market, of course contributes to their reported balance sheet size. However, the fourth column suggests otherwise. The continued significance of size confirms that money center banks are not only large, but with significant *other* business on their balance sheets. Economies of scale and scope both seem to play a role in a bank's network position. By including all three of the concepts in a single regression, it is clear that each explanatory variable is significant in concert with the others. In other words, systemic importance, network position, and crude asset size are all additive explanations for why a bank is in the core. Each concept adds a facet of core membership that is related to, but separate to the other two.

Much the same point is made in columns (5) through (7) of Table 1. This column utilizes definitions of bank size that are related to the size of interbank activity. Column (5) represents a regression which states that core membership is positively correlated to total interbank assets, although the same point could have been made with interbank liabilities. Column (6) states the same point with a variable, "interbank" which is the minimum of log assets and log interbank assets. The final regression looks at core membership as a function of network position, and interbank size, and finds that interbank size, in spite of the fact that it measures network activity, is not sufficient in explaining core membership. Both network position, as measured by betweeness, and network exposure, as measured by systemic importance, help to explain core membership, although systemic importance is significant only at the 8% level. All in all, the results of Table 1 suggest that money center banks are in the core because they are connected, not just in terms of their network position, as measured by betweeness, but also because of their systemic importance, and they are also in the core because of their ability to carry out large transactions, as measured either by total size, or by total interbank activity. Further, each of these concepts are not completely sufficient in and of themselves to explain core membership, but each add to the qualities that make up a money center bank.

3.2 Concluding remarks: bridging two literatures

In relating network position to bank-specific features our paper bridges two literatures. The banking literature, on the one hand, examines individual bank incentives without concern for how banks position themselves in a network. The literature on network formation, on the other hand, often relies on random processes from statistical mechanics (e.g. Newman et al (2006)). Even recent game-theoretic models of strategic network formation (Goyal (2007) and Jackson (2008) provide excellent surveys) disregard the attributes of individual nodes. In our view, this severely limits what such models can predict in the way of network formation. For instance, in some network formation games the pure star emerges as the unique equilibrium architecture (Bala and Goyal (2000), Goyal and Vega-Redondo (2007), Hojman and Szeidl (2008)); but since these theories cannot predict *which* node will form the center of the network, they must be regarded, in a sense, as indeterminate.

Our findings suggests that bank-specific attributes help explain how banks position themselves in the interbank market, as evidenced by the regression results of this section. As tiering is not random but behavioral, there are economic reasons why the banking system organizes itself around a core of money center banks. The strong correlation with size is indicative of the presence of fixed costs, or economies of scale and scope. To better understand financial networks, we argue that the way forward should focus more on the attributes of the nodes that make up the network. In the context of banking, this provides clues for theoretical modelling efforts as to how different banks choose to make network connections.

A class of recent banking models does take into account that interbank markets operate as networks rather than centralized exchanges. Allen and Gale (2000) proposed a framework in which banks of different regions (or sectors) face opposite liquidity shocks. This provides an incentive for banks to insure each other ex ante, which can be done through interbank deposits. (In a related model, Leitner (2005) demonstrates that interbank deposits help induce banks to bail each other out.) Similarly, Babus (2008) shows that it is optimal for banks to exchange deposits with all banks facing opposite liquidity shocks.[27] However, this approach predicts *dense* networks, contrary to the core-periphery structure we detected for the German interbank network. That structure is also highly *persistent*, which clashes with the view that unexpected liquidity shocks are the basis for understanding interbank activity. Moreover, the interbank market in these models is essentially *flat* – there is no role for intermediation. Banks are identical ex ante, including in the way they connect to each other. There is no reason in these models why banks, the main intermediaries in the economy, would build yet another layer of intermediation between them.

To explain the tiered structures we explored in this paper, a model would require some asym-

[27] It is unclear whether this theory predicts a network of interbank deposits. Other instruments are available for implementing risk-sharing including insurance contracts, derivatives, credit lines etc.

metry or specialization. Two existing models do so by assumption. In the two-tier bank model of Qi (2008), the central 'correspondent' bank is assumed to be different: its ability to borrow costlessly makes other banks use it as a liquidity pool, much like a central bank. However, central bank is not the only interbank intermediary, as is apparent from the German interbank network. Freixas et al (2000) provide an example of such a case, obtained by assuming that all travellers pass through a single location.[28] The bank located there receives and extends lines vis-à-vis banks in all other locations (which are not connected between each other).

Though these setting is constructed rather than derived, they lead to pure star networks with a single money center bank at its core. The core-periphery network is a generalization of the pure star with several interconnected centers. To better understand the formation of such networks, it would therefore seem promising to start out from a model featuring a variety of diverse banking firms.

[28] Consumers of different regions face uncertainty about where to consume. Interbank credit lines between banks in these regions help economize on reserves, so travellers need not move any goods or cash.

Appendix

Appendix A: Proofs

Part a) To show that the presence of intermediaries is necessary, consider a network N of dimension n in which the are *no* intermediaries in the sense of Definition 1. Banks are either lenders (λ in number), or borrowers (β in number), or neither of the two ($n - \lambda - \beta \geq 0$). We first show that the latter group, the unattached banks, must be in the periphery, because each unattached bank causes fewer errors in (3) relative to the model (2) when allocated to the periphery. To see this, suppose there is an unattached bank among the c banks in the core. This causes exactly $2(c-1)$ errors in the CC block, and $(n-c)$ errors in each of the blocks CP and PC of (3). The same bank placed in the periphery would cause no errors in CC (nor in PP), but could add up to $2(c-1)$ errors for expanding the CP and PC blocks (if all remaining core banks are not linked to the periphery). Switching the unattached bank from core to periphery thus leads to a net reduction in the total number of errors of *at least* $2(n-c)$, which is always positive (and zero if the periphery is empty). The move thus weakly dominates for the first unattached, and strictly dominates for each subsequent unattached bank and every combination of unattached banks. Therefore, it is optimal to allocate all unattached banks are in the periphery.

We proceed to show that the same argument holds for the remaining core banks, which must be either lenders or borrowers (not both). Suppose that λ_C lenders and β_C borrowers are in the core (so that $\lambda_C + \beta_C = c$, with $0 \leq \lambda_C \leq \lambda$, $0 \leq \beta_C \leq \beta$). Without loss of generality, reorder the nodes in each tier such that the lenders appear first, followed by the borrowers and the unattached. This divides each of the four blocks as shown in (7). The absence of intermediaries implies many zero blocks, since lenders borrow from no-one, borrowers lend to no-one, and the remaining banks are unattached. The non-zero entries show dimensions of sub-blocks that may be non-zero.

$$\begin{array}{cc|ccc} 0 & \lambda_C\beta_C & 0 & \lambda_C\beta_P & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & \lambda_P\beta_C & 0 & \lambda_P\beta_P & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \qquad (7)$$

Now, the number of errors of this (arbitrary) allocation can be reduced as long as there are banks left in the core. Applying (3) to (7) shows that the CC block generates at least $\lambda_C\left(\lambda_C - 1\right) + \beta_C\left(\beta_C - 1\right) + \lambda_C\beta_C$ errors, the number of zero entries in the top left block, and more if the sub-block $\lambda_C\beta_C$ is not complete with ones. The CP block (top right) comprises at least $\beta_C(n - \lambda_C -$

β_C) errors, where the term in brackets is the dimension of the periphery (of which $\beta_P \equiv \beta - \beta_C$ are borrowers). Likewise, the PC block counts at least $\lambda_C(n - \lambda_C - \beta_C)$ errors, and more if the sub-block $\lambda_P\beta_C$ is not column-regular as required by (2). This allocation thus produces, for these three blocks, at least

$$(n-1)(\lambda_C + \beta_C) + \lambda_C\beta_C \tag{8}$$

errors, plus the number of non-zeros in the sub-block $\lambda_P\beta_P$, denoted by $\#(\lambda_P\beta_P)$. If all banks were placed in the periphery instead, the errors equals the number of non-zeros, which cannot exceed $\lambda\beta$. Expanding $\lambda\beta$ (using $\lambda \equiv \lambda_C + \lambda_P$) shows that (8) exceeds $\#(\lambda\beta)$ provided

$$\lambda_C[(n-1) - \beta_P] + \beta_C[(n-1) - \lambda_P] > 0. \tag{9}$$

The terms in square brackets are always positive when there is one or more unattached banks in the network (implying $(n-1) > \beta + \lambda$); in that case, the error score can always be reduced by placing all banks in the periphery, i.e. until $\lambda_C = \beta_C = 0$. If there are no unattached banks, the same conclusion holds for all but one peculiar network for which the net gain in (9) would be zero.[29] Since moving all banks to the periphery is strictly dominant for all networks (and weakly dominant for one peculiar network), the absence of intermediaries implies an empty core.

To show sufficiency, i.e. that a network containing intermediaries gives rise to a non-empty core, assume to the contrary that the core is empty and at least one bank, say bank i, intermediates. Since all banks are in the periphery, the presence of i contributes at least two errors to PP. Allowing bank i to form a core by itself removes both errors without producing any new errors in the three new blocks of (3). By the same argument, adding more intermediaries to N can expand, but cannot reduce, the size of the core. Thus the presence of intermediaries produces a core.

What remains to be checked is that the periphery does not vanish. The core is potentially largest when all banks lend to each other: placing $n-1$ banks in the core will minimize the error score to zero. The same score can be also attained by moving all n banks to the core, which would leave no periphery. However, one missing bilateral link is sufficient (not necessary) to guarantee that a periphery always exists. Suppose banks i and j are not connected to each other ($N_{ij} = N_{ji} = 0$). The two zeros contribute two errors in CC if both banks remain in the core. Moving i or j jointly to the periphery yields a net gain: the two zeros are now in the PP block where they do not count as errors, and the CP and PC blocks that this move created

[29] If a *single* bank lends to *all* other banks in the system ($\beta_P = n-1$, and $\beta_C = 0$), then the total error score is unaffected by whether that lender is in the core or in the periphery. (The same holds for the single-borrower case, where $\lambda_P = n-1$, and $\lambda_C = 0$.)

cannot contain more errors than they did as part of the CC block.[30] A single missing link is therefore sufficient to sustain a periphery even when all other banks lend to each other.

Part b) The proof that all core banks are intermediaries is by contradiction. Suppose a bank that does *not* intermediate is in the core. We show that the distance-minimizing procedure will place this bank in the periphery. A bank that does not intermediate has no out-going interbank links, or no incoming links, or no links at all. We only need to consider one case, that of zero out–degree.[31] First compute how many errors this bank, say i, causes as a member of the core. The core consists of c banks including i, and we use (3) to aggregate errors in the four blocks delineated by the single lines in the matrix below. Links with core banks never cause errors, so we can focus on the missing links. By not lending at all, bank i contributes at least $(c-1)$ errors to CC, plus $(n-c)$ errors to CP for violating row-regularity in that block. This contribution to the error score, $n-1$, is a *minimum* value: it is higher if bank i does not borrow from *all* other core banks, or if it does not borrow from the periphery.

CC	0	CP
	1	
0 0	0	0 0
	0	
PC	1	PP

Moving bank i to the periphery will permit a net reduction in the number of errors. This move changes the four blocks as indicated by the *double* lines in the matrix. The CC block shrinks, transferring its column i to CP and row i to PC, respectively; and PP expands, taking column i from PC and row i from CP, respectively. The first transfer removes all the errors that i had caused in CC, and may add new errors to CP and PC that are strictly fewer in number than those saved CC. (There is one possible exception where the net gain reaches zero. This occurs only if none of the remaining core banks borrow from any periphery banks ($c-1$ errors), and either some core banks do not lend to the periphery, or bank i borrows from all core banks.) The second transfer also delivers a net improvement: the $(n-c)$ errors formerly in CP no longer count as errors when moved to PP, but column i now in PP may add errors if it contains ones; the net reduction in errors is again strictly positive, except in the one case

[30] If each core bank is connected to at least one bank among i and j, the new CP and PC blocks will contain no errors at all. If some core banks are attached to neither i nor j, then the corresponding rows in CP (columns in PC) will contain as many errors as was the case when these rows (columns) were part of the CC block. This continues to hold even if all core banks are unconnected to this pair of banks (then i and j are unattached and best put in the periphery, as shown above). Moving i and j to the periphery saves at least two errors in each case.

[31] The case of zero in–degree is symmetric. That unattached banks go to the periphery was shown in part a).

where bank i happens to borrow from *all* $(n-c)$ banks in the periphery.

Combining these error reductions shows that the distance-minimizing procedure will move bank i to the periphery, contradicting the initial claim that a non-intermediary can be in the core. (The one exception for which there is weak dominance can occur only if i borrows from all banks, or some other core banks do not intermediate between periphery banks, a case considered in what follows.) Thus all core banks are intermediaries.

The converse, that all intermediaries are also core banks, does not hold. Suppose bank i is in the core but intermediates only among core banks. It is straighforward to show, with the approach just used, that moving i to the periphery always produces a net reduction of at least $2\,(n-c)$ errors (which had been in CP and PC but no longer count as errors when part of PP). Hence, not every intermediary is a core bank.

We generalize this case by showing that a core bank that does not lend to (*or* does not borrow from) the periphery will not be in the core. Suppose bank i does not lend to any bank in the periphery. Its presence in the core contributes $(n-c)$ errors to CP, and $x \geq 0$ errors to CC for any missing links with other core banks. Moving bank i to the periphery again leads to a net reduction in errors. The argument follows exactly the one just advanced for non-intermediaries, the only difference being that the number of errors involved in the first transfer, now x, need not exceed $(c-1)$. The result carries through that moving i to the periphery is strictly dominant, again with one exception where it is weakly dominant. The analogous case of a bank that does not *borrow* from the periphery can be shown by symmetry. Therefore, the core excludes intermediaries that do not lend to (or do not borrow from) the periphery.

Appendix B: Computational methods

As stated, fitting a core-periphery model to a real-world network is a large-scale problem in combinatorial optimization which we solve by means of a sequential algorithm. This way, the search for the optimal core leads to a solution in polynomial time, rather than in exponential time (2^n) required by exhaustive search. Section 1.3 described two versions of the algorithm that we designed for this task, both running in polynomial time (order n^1).[32] In our application to the German network ($n = 1802$), the algorithm converged in 70 seconds on a standard IntelCore 2 duo processor (2.4GHz).

For NP-hard problems of this dimension, it is not possible to prove that the solution returned by any procedure is indeed the global optimum. We therefore performed several robustness checks to dispel doubts. First, we backtested our algorithm against existing blockmodeling

[32] The MATLAB code is available upon request from the authors.

routines, and obtained the same solutions for small example networks.[33] We also tested that the algorithm finds the optimum for cases where the true solution is known: we generated artificial networks (of the same dimension and density as the German system) with a *perfectly* tiered structure, for which the minimum error score (4) must be zero, by construction. The algorithm consistenly returned the correct set of core banks with zero errors. Second, we know from Proposition 1 that any solution returning non-zero elements on the off-diagonal of the error matrix E cannot be an optimum – in practice, the procedure never returned solutions failing this criterion. However, as this is a necessary (not a sufficient) condition, one cannot rely on this test alone to rule out all local optima. Our third and main robustness check therefore consisted of repeated application and careful comparision of the results generated by two algorithms (see section 1.3).

This was straightforward to do for the single application to the German interbank network, and reliably yielded the solution reported in the text. To prepare the thousands of runs necessary for hypothesis testing, we compared the error scores calculated with simulated annealing programs with various "cooling" parameters and many different initial partitions, with the greedy algorithms using different initial conditions. For avoiding local optima it turned out to be helpful to start the greedy algorithm sufficiently far from an approximate solution to give it time to converge to the error-minimizing core. The best simulated annealing algorithms gave error scores very close to a greedy algorithm with initial partitions assigning a random half of banks to the core. The local optima that did occur were easily identified by their extremely high error score, which would fall to the normal range when fitting the same network again.

The distributions shown in Figure 6, using the greedy algorithm with random initial partitions, offered consistently the minimum error score, always close to the best solution of any of the algorithms we tried. We performed robustness checks on the algorithm to make sure that the initial conditions and parameters were consistent with generating the minimum error scores both types of random networks (see Appendix B). The core sizes did not vary between the algorithms, and although the error scores did fluctuate in a narrow range for different initial conditions. Taken together, these robustness tests assured us that the distributions generated for the hypothesis tests reflect the intrinsic randomness of random networks, rather than stochastic output from an unreliable procedure.

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Figures and Table

Figure 1: Illustration of Interbank Tiering

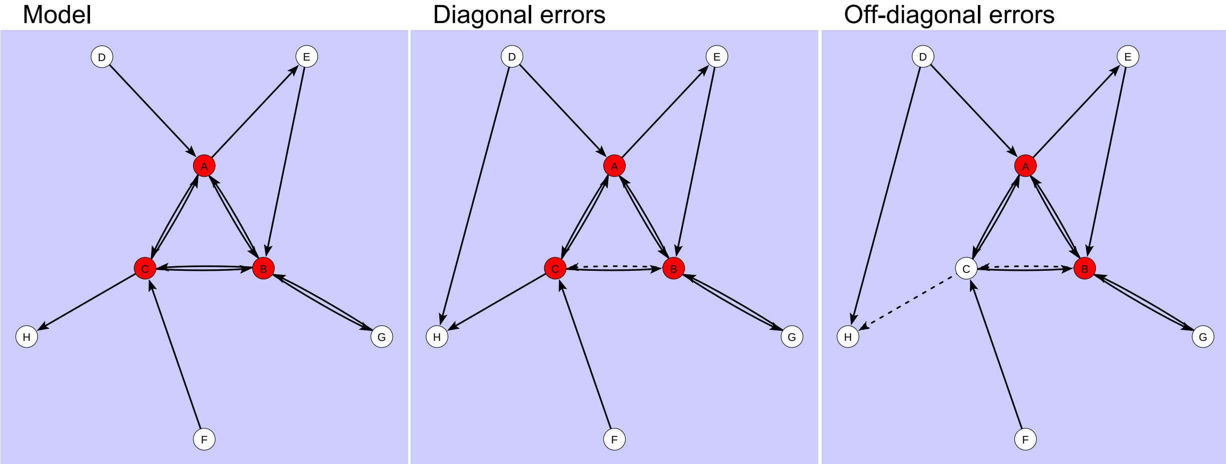

Figure 2: Tiering as a refinement of intermediation

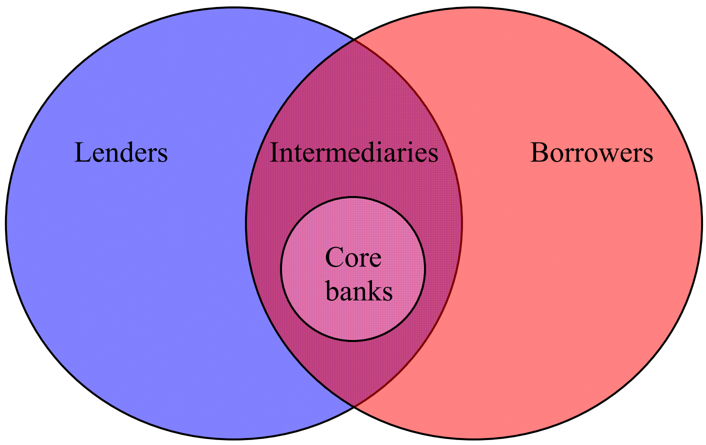

Figure 3: Structural stability over time

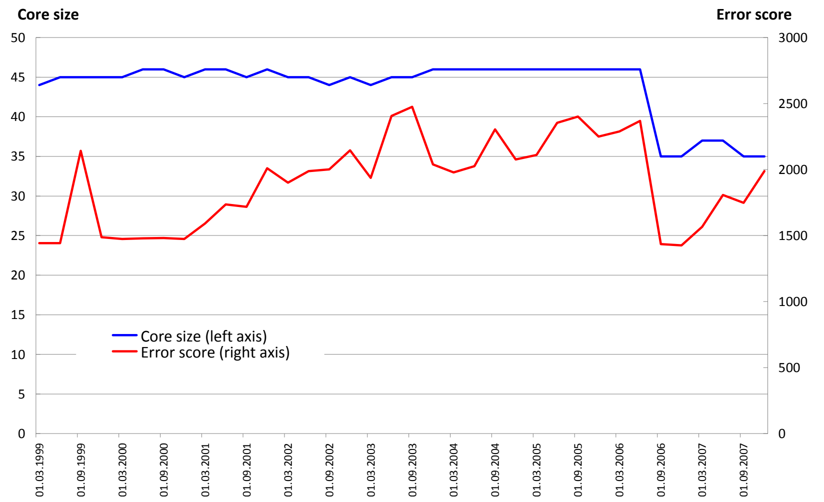

Figure 5: Robustness

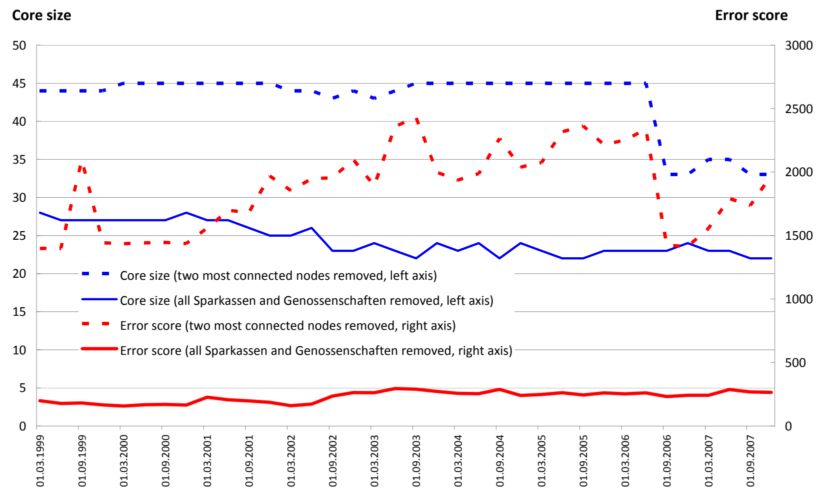

Figure 4: Transition probabilities over time

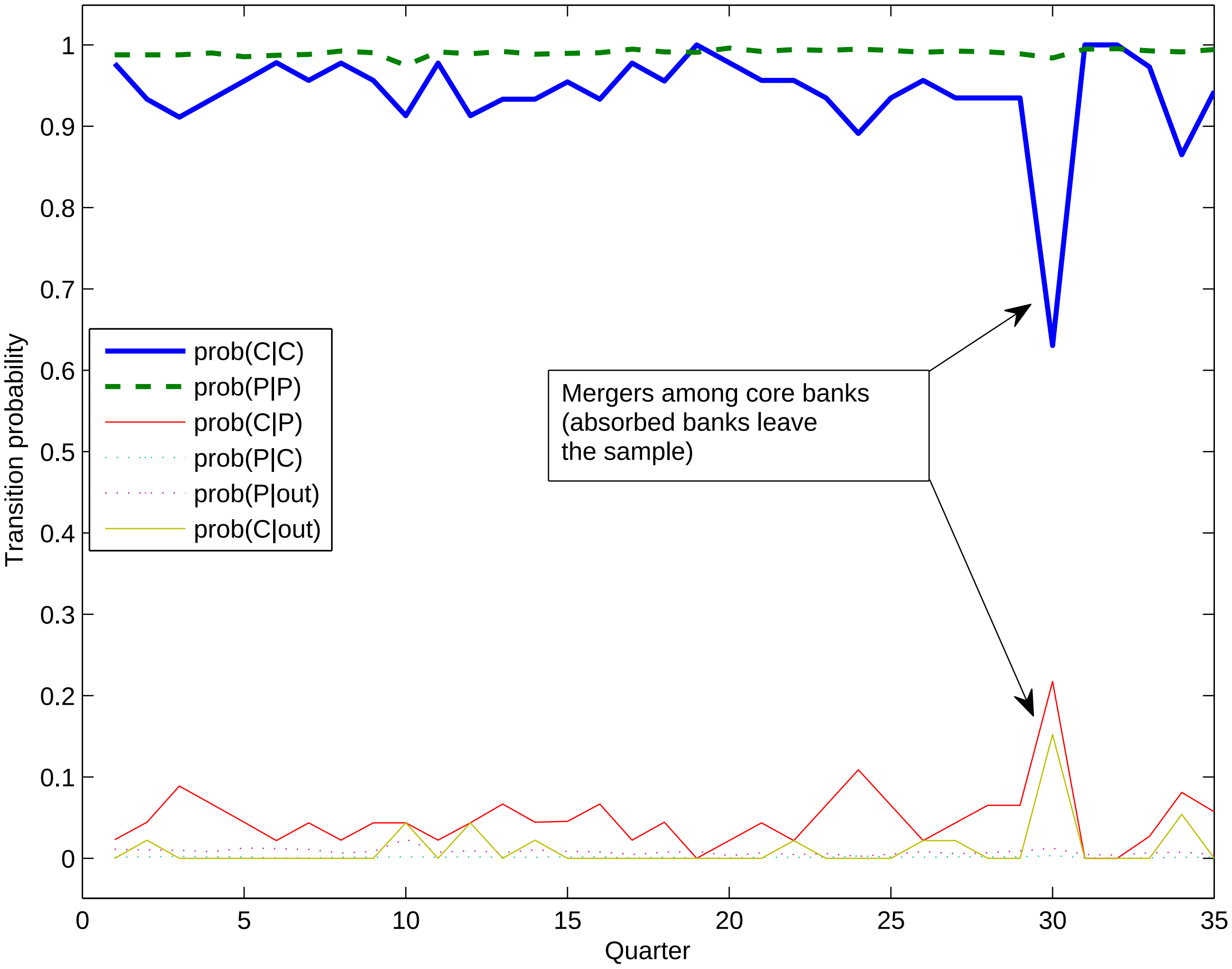

Figure 6: Test against error distributions on random networks

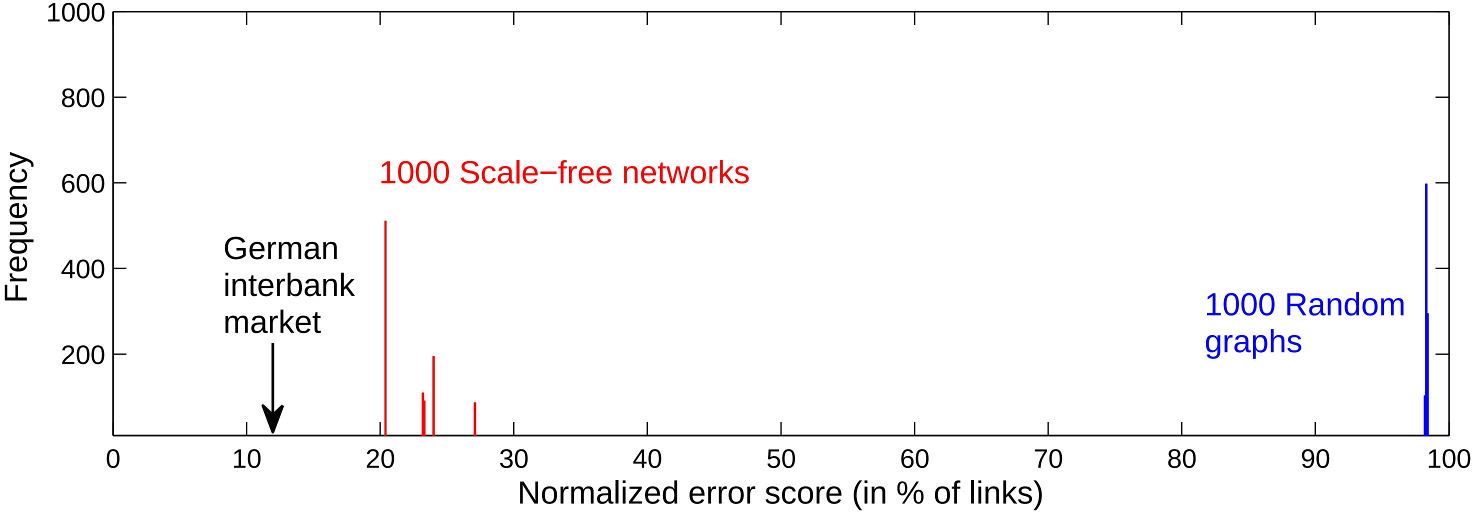

Table 1: Core membership and bank-specific variables

Regressor	*(1)*	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*
Balance sheet size[1]	0.903**			0.361**			
Interbank liabilities[2]					0.667**		
Interbank intermediation[3]						0.718**	0.455**
Systemic importance[4]		4.737**		3.292*			2.206
Betweenness[5]			3962**	2931**			3393**
Pseudo-R2	0.573	0.475	0.654	0.736	0.542	0.579	0.765
% correctly classified[6]	98.5%	98.5%	98.8%	99.0%	98.0%	98.7%	99.1%
Prob(c|C)[6] idf core correct	48.9%	42.2%	60.0%	68.9%	42.2%	51.1%	71.1%
Prob(c|P)[6] idf core false	0.17%	0.06%	0.17%	0.17%	0.57%	0.06%	0.23%
Number of observations	1779	1802	1802	1779	1802	1802	1802

Notes: The table presents the results from Probit regressions (shaded variables rely on full network data). The discrete variable "core membership" (taking the value 1 for core banks, 0 otherwise) is regressed on a constant and the variables shown in the columns. Significance is shown as *(5%) and **(1%). [1] Natural logarithm of total assets (in € thousands). [2] Log (interbank liabilities+1). The fit with interbank liabilities was slightly better that that with interbank assets (not reported). [3] The log of intermediated interbank positions, Log(min{interbank assets, interbank liabilities}+1). [4] The systemic importance of an institution is measured here as the (inverse) loss-given-default necessary such that the failure of the institution leads to a systemic crisis (a quarter of the banking system in default). [5] Normalized betweenness centrality (Freeman (1979)). [6] Probabilities are evaluated at default threshold of 0.5. Prob(c|C) = probability (in %) that a bank predicted to be in the core is indeed in the core (=100-Prob(P|C)). Prob(c|P) = rate of false core predictions. Prob(c|C) can be raised by lowering the threshold.